

Funkcije več spremenljivk

Limita in zveznost

1. Izračunaj naslednje limite

(a) $\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 - y^2}{x^2 + y^2}$

(b) $\lim_{(x,y) \rightarrow (0,0)} \frac{2xy}{x^2 + y^2}$

(c) $\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 + y^2}{\sqrt{x^2 + y^2 + 1} - 1}$

(d) $\lim_{(x,y) \rightarrow (0,0)} \frac{xy^2}{x^2 + y^2}$

(e) $\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 y^2}{(x^2 + y^2)^2}$

(f) $\lim_{(x,y) \rightarrow (1,0)} \frac{(x-1)^2 \ln x}{(x-1)^2 + y^2}$

Rešitev:

(a) Ne obstaja.

(b) Ne obstaja.

(c) 2

(d) 0

(e) Ne obstaja.

(f) 0

2. Določi število d tako, da bo funkcija

$$f(x, y) = \begin{cases} \frac{xy \sin(x^2 + y^2)}{(x^2 + y^2)^{\frac{3}{2}}} & : (x, y) \neq (0, 0) \\ d & : (x, y) = (0, 0) \end{cases}$$

zvezna.

Rešitev: $d = 0$

3. Ali se dajo naslednje funkcije zvezno razširiti v izhodišče?

(a) $f(x, y) = \frac{x^3 + x^2 + y^2}{x^2 + y^2}$

(b) $f(x, y) = \frac{x^2+2y^2}{2x^2+y^2}$

Rešitev:

(a)

$$f(x, y) = \begin{cases} \frac{x^3+x^2+y^2}{x^2+y^2} & : (x, y) \neq (0, 0) \\ 1 & : (x, y) = (0, 0) \end{cases}$$

(b) Zvezna razširitev ne obstaja.

Parcialni odvod

4. Izračunajte parcialne odvode prvega in drugega reda naslednjih funkcij:

(a) $f(x, y) = x^3y^2 + 4x^2 + 2y^5$,

(b) $f(x, y) = x^y$

(c) $f(x, y) = \frac{x}{y}$

Rešitev:

(a) $\frac{\partial f}{\partial x} = 3x^2y^2 + 8x$, $\frac{\partial f}{\partial y} = 2x^3y + 10y^4$,

$$\frac{\partial^2 f}{\partial x^2} = 6xy^2 + 8, \frac{\partial^2 f}{\partial y^2} = 2x^3 + 40y^3, \frac{\partial^2 f}{\partial x \partial y} = 6x^2y$$

(b) $\frac{\partial f}{\partial x} = yx^{y-1}$, $\frac{\partial f}{\partial y} = x^y \ln x$,

$$\frac{\partial^2 f}{\partial x^2} = y(y-1)x^{y-2}, \frac{\partial^2 f}{\partial y^2} = x^y (\ln x)^2, \frac{\partial^2 f}{\partial x \partial y} = x^{y-1} + yx^{y-1} \ln x$$

(c) $\frac{\partial f}{\partial x} = \frac{1}{y}$, $\frac{\partial f}{\partial y} = -\frac{x}{y^2}$,

$$\frac{\partial^2 f}{\partial x^2} = 0, \frac{\partial^2 f}{\partial y^2} = \frac{2x}{y^3}, \frac{\partial^2 f}{\partial x \partial y} = -\frac{1}{y^2}$$

5. Naj bo $f(x, y) = \frac{x \cos y - y \cos x}{1 + \sin x + \sin y}$. Izračunaj $\frac{\partial f}{\partial x}(0, 0)$ in $\frac{\partial f}{\partial y}(0, 0)$.

Rešitev: $\frac{\partial f}{\partial x}(0, 0) = 1$, $\frac{\partial f}{\partial y}(0, 0) = -1$

6. Poišči vrednost izraza $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$, če je

$$f(x, y) = \frac{1}{\sqrt{x^2 + y^2 + z^2}}.$$

Rešitev: 0