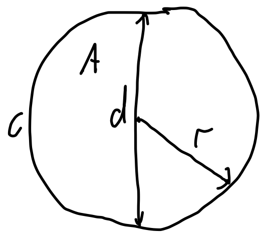
The most famous real number defined as a ratio. (Last week's notes)



d = diameter

C = circumference

r = radius

A = area

The ratio $c:d$ is the same irrespective of the size of the circle.

This defines the real number pi (don't confuse with pie)

$$\pi = \frac{c}{d}$$

π occurs ubiquitously in mathematics; eg,

By definition: $C = \pi d = 2\pi r$ circumference

$A = \pi r^2$ area of the circle

$V = \frac{4}{3}\pi r^3$ volume of a sphere

$4\pi r^2$ surface area of a sphere

Watch the video: " .. area of a circle"

- where is the speaker from? (Australia)
- 2.58 "from thin air"
- 3.28 "concentric"
- 4.59 "unfurl"

what do these mean?

- At 6.12 what is the mistake in the subtitles?

Subtitles: "I know what it's faces"

Spoken: "I know what its base is"

We're familiar with π

$$\pi = 3.14159265 35 \ldots$$

which means

$$\pi = 3 + \frac{1}{10} + \frac{4}{100} + \frac{1}{1000} + \frac{5}{10^4} + \frac{9}{10^5} \ldots$$

This <u>infinite series</u> sums to precisely π

There is no obvious pattern to π.

However there are other infinite series for π that <u>do</u> have obvious patterns.

e.g.,

$$\pi = \frac{4}{1} - \frac{4}{3} + \frac{4}{5} - \frac{4}{7} + \frac{4}{9} - \frac{4}{11} + \cdots$$

(The Gregory - Leibniz series.)

The G-L series converges too slowly to be useful for calculating π.

But there are other series that are more efficient and are used for such calculations.

Calculating π is nowadays a benchmark for high-performance computing.

The new world record August 2021

62,831,853,071,796 digits

(The University of Applied Sciences of Eastern Switzerland)

Another large number

299,792,438

The speed of light in m/s.

This number is exactly the speed of light because the metre is defined in terms of the speed of light. (1983)!

A very small number from science

$$0.\underbrace{00000\ldots000}_{30\text{ zeros}}9109\ldots$$

The mass of an electron in kg
(not an exact number)

We typically write large and small numbers using <u>scientific notation</u>

e.g.

2.99792458×10^{8} m/s

3.00×10^{8} m/s (3 sig. figures)

9.109×10^{-31} kg (4 sig. figures)

Scientific notation

We write a number as

$$m \times 10^{n}$$

the mantissa or significand

exponent

m is a real number

n is an integer

The number $m \times 10^n$ is in standard form if $1 \leq m < 10$ (normalised form)

We often write such numbers to a specified number of significant figures

(significant digits)

It is easy to perform arithmetic calculations on numbers in scientific notation

For example to multiply

$$(m_1 \times 10^{n_1}) \times (m_2 \times 10^{n_2})$$

we calculate

$$(m_1 \times m_2) \times 10^{(n_1 + n_2)}$$

To multiply numbers in scientific notation, we multiply the significands and add the exponents.

How does does one do addition?

Computers use a version of scientific notation to represent real numbers called <u>floating point</u>.

This is based on scientific notation in <u>binary</u>.

E.g.

$$\begin{array}{ccccccccc} 2^2 & 2^1 & 2^0 & & 2^{-1} & 2^{-2} & 2^{-3} & 2^{-4} \\ 1 & 0 & 1 & . & 1 & 0 & 0 & 1 \\ 10^2 & 10 & 10^0 & & 10^{-1} & 10^{-2} & 10^{-3} & 10^{-4} \end{array}$$

in base 10 $= 100 + 1 + \frac{1}{10} + \frac{1}{10000}$

in binary $= 4 + 1 + \frac{1}{2} + \frac{1}{16} = 5\frac{9}{16}$

This is binary notation with a "binary point"

For floating point watch video
"floating point numbers"