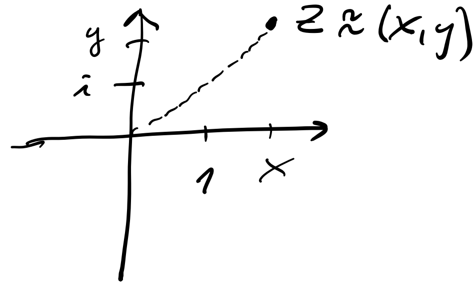
KOMPLEKSNA ŠTEVILA

$z = x + iy \in \mathbb{C}$



$x = \mathrm{Re}(z) \in \mathbb{R}$ REALNI DEL

$y = \mathrm{Im}(z) \in \mathbb{R}$ IMAGINARNI DEL

$i = \sqrt{-1}$

OPERACIJE:

~ $\bar{z} = \overline{x+iy} = x - iy$ KONJUGACIJA

~ $|z| = \sqrt{x^2+y^2} = \sqrt{z \cdot \bar{z}}$ ABS. VREDNOST

~ SEŠTEVANJE, MNOŽENJE, DELJENJE 'PO KOMPONENTAH'

1. (a) $z^2 + 2i\,\mathrm{Re}(z) = |z|$

   (b) $|z+1| = |2z-1|$

   REŠI ENAČBI V OBSEGU KOMPLEKSNIH ŠTEVIL, TER MNOŽICO REŠITEV NARIŠI.

(a) $z = x + iy$

$$z^2 = (x+iy)^2 = x^2 + 2ixy + (iy)^2 = x^2 + 2ixy - y^2$$

$$2i\,\mathrm{Re}(z) = 2ix$$

$$|z| = \sqrt{x^2+y^2}$$

$$x^2 + 2ixy - y^2 + 2ix = \sqrt{x^2+y^2}$$

DOBIMO SISTEM ENAČB:

$$x^2 - y^2 = \sqrt{x^2+y^2}$$

$$2xy + 2x = 0 = 2x(y+1)$$

(i) $x = 0 \Rightarrow -y^2 = \sqrt{y^2} \Rightarrow y = 0$

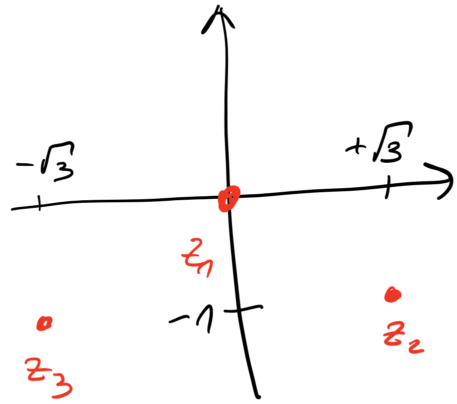
$$\boxed{z_1 = 0 + i0 = 0}$$

(ii) $y=-1 \Rightarrow \boxed{x^2-1=\sqrt{x^2+1}} \;/^2$

$$x^4-2x^2+\cancel{1}=x^2+\cancel{1}$$
$$x^4-3x^2=0$$
$$x^2(x^2-3)=0$$
$$\cancel{x_1=0} \qquad x_{2,3}=\pm\sqrt{3}$$

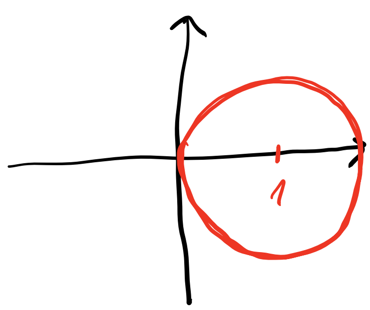
NE USTREZA
ORIG. ENAČBI

$$\boxed{z_{2,3}=\pm\sqrt{3}-i}$$



(b) $|z+1|=|2z-1|$

$$z=x+iy$$
$$|x+iy+1|=|2(x+iy)-1|$$
$$|(x+1)+iy|=|(2x-1)+2iy|$$
$$\sqrt{(x+1)^2+y^2}=\sqrt{(2x-1)^2+4y^2} \;/^2$$
$$x^2+2x+\cancel{1}+y^2=4x^2-4x+\cancel{1}+4y^2$$
$$0=3x^2-6x+3y^2 \;/:3$$
$$0=x^2-2x+y^2$$
$$0=(x-1)^2-1+y^2$$
$$1=(x-1)^2+y^2$$



ENAČBO REŠIJO VSA ŠTEVILA
NA KROŽNICI S POLMEROM 1
IN SREDIŠČEM V $(1,0)$ OZ. $z=1$.

② SKICIRAJ PODMNOŽICE KOMPLEKSNIH ŠTEVIL.

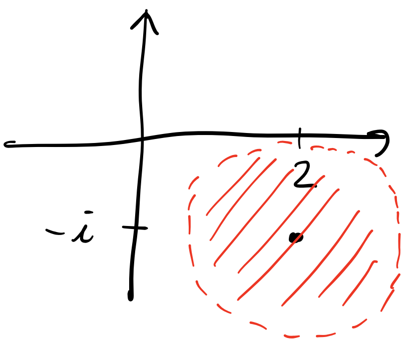
(a) $\{z \in \mathbb{C} : |z-2+i| < 1\}$

(b) $\{z \in \mathbb{C} : |z-1| = |z-i|\}$

(c) $\{z \in \mathbb{C} : |z| + |z-3| = 4\}$

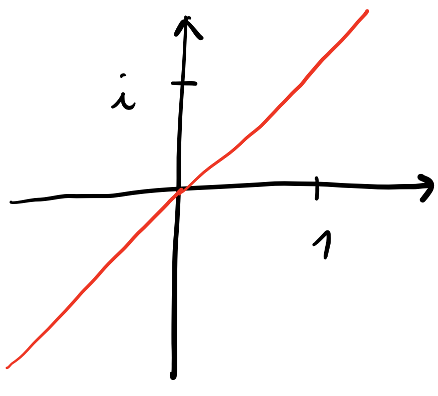
NAPRAVI TUDI RAČUNSKI PREIZKUS.

(a) GRE ZA TOČKE, KI SO OD $z = 2-i$ ODDALJENE ZA MANJ KOT 1.



$$|z-2+i| < 1$$
$$|x+iy-2+i| < 1$$
$$|(x-2)+i(y+1)| < 1$$
$$\sqrt{(x-2)^2+(y+1)^2} < 1 \quad /^2$$
$$\underbrace{(x-2)^2+(y+1)^2 < 1}_{\text{NEENAČBA ZA ODPRTI KROG}}$$

(b) $|z-1| = |z-i|$

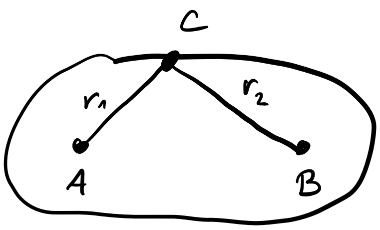


IŠČEMO MNOŽICO TOČK, KI SO ENAKO ODDALJENE OD i IN 1 $\Rightarrow$ SIMETRALO MED NJIMA.

$$|x+iy-1| = |x+iy-i|$$
$$|(x-1)+iy| = |x+i(y-1)|$$
$$\sqrt{(x-1)^2+y^2} = \sqrt{x^2+(y-1)^2} \quad /^2$$
$$\cancel{x^2}-2x+\cancel{1}+\cancel{y^2} = \cancel{x^2}+\cancel{y^2}-2y+\cancel{1}$$
$$-2x = -2y$$
$$x = y$$

SIMETRALA LIHIH KVADRANTOV.

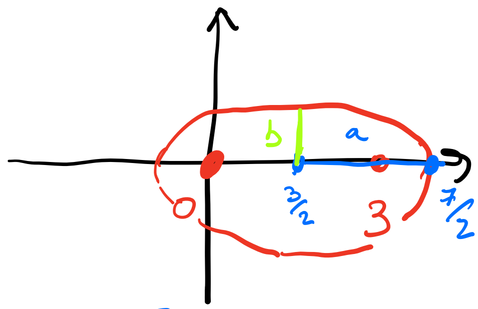
(c) $|z| + |z-3| = 4$



$r_1 + r_2 = \text{konst.}$

DEFINICIJA ELIPSE

MI IMAMO ELIPSO Z GORIŠČI V $z=0$ IN $z=3$, TER VSOTO RAZDALJ ENAKO 4.



$a=2, \; e=\frac{3}{2}$

$b = \sqrt{a^2-e^2} = \sqrt{4-\frac{9}{4}} = \sqrt{\frac{7}{4}} = \frac{\sqrt{7}}{2}$

ELIPSA S SREDIŠČEM V $(\frac{3}{2},0)$ IN POLOSEMA $a=2$ IN $b=\frac{\sqrt{7}}{2}$.

NAREDIMO ŠE RAČUNSKI PREIZKUS:

$$|z| + |z-3| = 4$$

$$\sqrt{x^2+y^2} + \sqrt{(x-3)^2+y^2} = 4$$

$$\sqrt{(x-3)^2+y^2} = 4 - \sqrt{x^2+y^2} \quad /^2$$

$$\cancel{x^2} - 6x + 9 + \cancel{y^2} = 16 - 8\sqrt{x^2+y^2} + \cancel{x^2} + \cancel{y^2}$$

$$8\sqrt{x^2+y^2} = 7 + 6x \quad /^2$$

$$64x^2 + 64y^2 = 36x^2 + 84x + 49$$

$$28x^2 - 84x + 64y^2 = 49$$

ČEMU ŽELIMO, DA JE TO ENAKO?

$$\underbrace{1 = \frac{(x-\frac{3}{2})^2}{4} + \frac{y^2}{\frac{7}{4}}}_{\text{ENAČBA NAŠE ELIPSE}} \quad / \cdot 4 \cdot \frac{7}{4}$$

$$7 = \frac{7}{4}\left(x - \frac{3}{2}\right)^2 + 4y^2$$

$$7 = \frac{7}{4}\left(x^2 - 3x + \frac{9}{4}\right) + 4y^2$$

$$7 = \frac{7}{4}x^2 - \frac{21}{4}x + \frac{63}{16} + 4y^2 \quad / \cdot 16$$

$$112 = 28x^2 - 84x + 63 + 64y^2$$

$$49 = 28x^2 - 84x + 64y^2$$

RES DOBIMO ENAKO ZVEZO.

(3) NAJ BO $|z| = 1$ IN $z \neq 1$. DOKAŽI, DA JE ŠTEVILO $w = i\,\frac{z+1}{z-1}$ REALNO.

$z = x + iy$

$$w = i\,\frac{x+iy+1}{x+iy-1} = i\,\frac{(x+1)+iy}{(x-1)+iy} \cdot \frac{(x-1)-iy}{(x-1)-iy} =$$

$$= i\,\frac{(x^2-1) + iy(x-1) - iy(x+1) - (iy)^2}{(x-1)^2 - (iy)^2} =$$

$$= i\,\frac{x^2 - 1 - 2iy + y^2}{(x-1)^2 + y^2} = i\,\frac{-2iy}{(x-1)^2+y^2} = \frac{2y}{(x-1)^2+y^2} \in \mathbb{R}$$

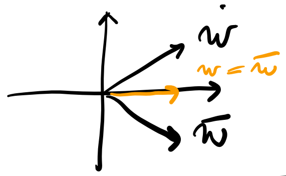
$x^2 + y^2 = 1$

KER $|z| = 1$

BOLJ ELEGANTNA REŠITEV:

UPORABIMO DEJSTVO:

$$\boxed{w \in \mathbb{R} \iff w = \overline{w}}$$



VELJA TUDI:

- $\overline{z_1 \cdot z_2} = \overline{z_1} \cdot \overline{z_2}$
- $\overline{z_1 + z_2} = \overline{z_1} + \overline{z_2}$
- $\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\overline{z_1}}{\overline{z_2}}$, ČE $z_2 \neq 0$.

$$\overline{w} = \overline{i \, \frac{z+1}{z-1}} = \overline{i} \cdot \frac{\overline{z+1}}{\overline{z-1}} = -i \cdot \frac{\overline{z}+1}{\overline{z}-1}$$

VEMO PA TUDI, DA JE $|z| = 1 \Rightarrow \sqrt{z\overline{z}} = 1$

$\Rightarrow z\overline{z} = 1$

$\Rightarrow \overline{z} = \frac{1}{z}$

$$\overline{w} = -i \cdot \frac{\frac{1}{z} + 1 \quad / \cdot z}{\frac{1}{z} - 1 \quad / \cdot z} = -i \, \frac{1+z}{1-z} = i \, \frac{z+1}{z-1} = w$$

TOREJ ZA $|z| = 1$ IN $z \neq 1$ VELJA $w = \overline{w}$ t.j. $w \in \mathbb{R}$.