Rešitve
kolokvija 1
MAT 1

① $6 \mid n^3 + 11n$

INDUKCIJA:

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$n=1: n^3+11n = 12$ je deljivo s 6 ✓

$n \to n+1$

$(n+1)^3 + 11(n+1) = n^3 + 3n^2 + 3n + 1$
$+ 11n + 11$

$= \underbrace{n^3 + 11n}_{\text{deljivo s 6 po I.P.}} + 3n^2 + 3n + 12$

$= \underbrace{6k + 3\underbrace{n(n+1)}_{\text{deljivo z 2}} + 12}_{\text{deljivo s 6}}$ 5

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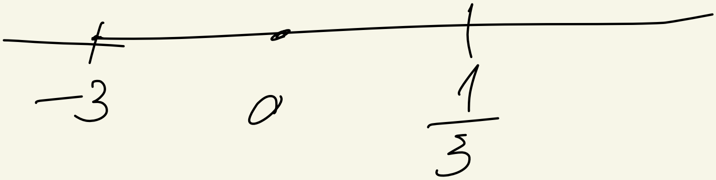
vsak člen deljiv s 6.

(2) $|x^2+3x| - 2x \le |1-3x| + 4$

$x(x+3)$



1.) $x < -3$

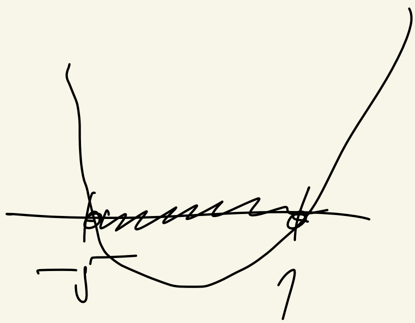
$(-x)(-x-3) - 2x \le 1 - 3x + 4$

$x^2 + 3x - 2x \le 5 - 3x$

$x^2 + 4x - 5 \le 0$

$(x+5)(x-1) \le 0$

$x_1 = -5 \quad x_2 = 1$



$\cap$

$x < -3$

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$R: [-5, -3)$

2.) $x \in [-3, 0)$

$$-x(x+3) - 2x \leq 1 - 3x + 4$$

$$-x^2 - 3x - 2x \leq 5 - 3x$$

$$-x^2 - 5x \leq 5 - 3x$$

$$0 \leq x^2 + 2x + 5$$

$$D = b^2 - 4ac = 4 - 20 < 0$$

6

R: $x \in [-3, 0)$

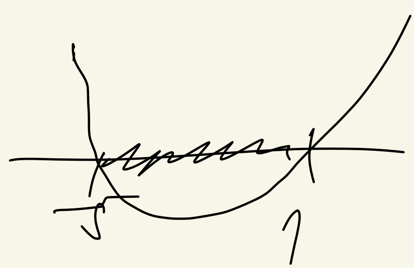
3.) $x \in [0, \frac{1}{3})$

$$x(x+3) - 2x \leq 1 - 3x + 4$$

$$x^2 + 3x - 2x \leq 5 - 3x$$

$$x^2 + 4x - 5 \leq 0$$

$$(x+5)(x-1)$$



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R: $\cap [0, \frac{1}{3}) = [0, \frac{1}{3})$

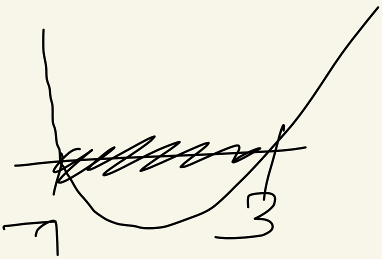
4.) $x \geq \frac{1}{3}$

$x(x+3) - 2x \leq 3x - 1 + 4$ 6

$x^2 + x \leq 3x + 3$

$x^2 - 2x - 3 \leq 0$

$(x-3)(x+1) \leq 0$



$[\frac{1}{3}, \infty) = [\frac{1}{3}, 3]$

R = unija svih intervala 1

$= [-5, 3]$

3) $w = \frac{a+i}{a-i} \cdot \frac{(a+i)}{(a+i)}$

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$$= \frac{a^2 + 2ai - 1}{a^2+1} = \underbrace{\frac{a^2-1}{a^2+1}}_{Re(w)} + i \underbrace{\frac{2a}{a^2+1}}_{Im(w)}$$

$$|w| = \sqrt{\frac{(a^2-1)^2}{(a^2+1)^2} + \frac{4a^2}{(a^2+1)^2}}$$ 3

$$= \sqrt{\frac{a^4 - 2a^2 + 1 + 4a^2}{(a^2+1)^2}} = \sqrt{\frac{\cancel{(a^2+1)^2}}{\cancel{(a^2+1)^2}}}$$

$$= 1$$

rešitev naloge:

$$z^5 - 3z^4w - zw + 3w^2 = 0$$

$$z(z^4 - w) - 3w(z^4 - w) = 0$$

(3)

$$(z - 3w)(z^4 - w) = 0$$

$$z_1 = 3w \qquad z_2 = \sqrt[4]{w}$$

a) b)

za $a = \frac{1}{\sqrt{3}}$ je

$$w = \frac{\frac{1}{3} - 1}{\frac{1}{3} + 1} + i\,\frac{\frac{2}{\sqrt{3}}}{\frac{1}{3} + 1} = \frac{-\frac{2}{3}}{\frac{4}{3}} + i\,\frac{\frac{\cancel{2}}{\sqrt{3}}}{\frac{\cancel{4}\,2}{3}}$$

(3)

$$= -\frac{1}{2} + i\,\frac{3}{2\sqrt{3}} = \underline{-\frac{1}{2} + i\,\frac{\sqrt{3}}{2}}$$

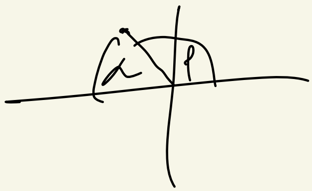
$$z_1 = 3w = -\frac{3}{2} + \frac{i\,3\sqrt{3}}{2}$$ (3)

Uwaga: $z^4 = -\frac{1}{2} + i\frac{\sqrt{3}}{2}$

✓ połowa dobrze

$$r^2 = \frac{1}{4} + \frac{3}{4} = 1 \Rightarrow r = 1$$

$$\tan\alpha = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3} \Rightarrow \alpha = \frac{\pi}{3}$$



połowa (3)

$$\Rightarrow z^4 = \cos\left(\frac{2\pi}{3}\right) + i\sin\left(\frac{2\pi}{3}\right)$$

$$z_{2,3,4,5} = \cos\left(\frac{\frac{2\pi}{3} + 2k\pi}{4}\right) + i\sin\left(\frac{\frac{2\pi}{3} + 2k\pi}{4}\right)$$

$$k = 0, 1, 2, 3$$

$$z_2 = \cos\left(\frac{\pi}{6}\right) + i\sin\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2} + i\frac{1}{2}$$

$$z_3 : \text{kat} = \frac{\frac{2\pi}{3} + 2\pi}{4} = \frac{\pi}{6} + \frac{\pi}{2} = \frac{2\pi}{3}$$

$$\Rightarrow z_3 = \cos\left(\frac{2\pi}{3}\right) + i\sin\left(\frac{2\pi}{3}\right) = -\frac{1}{2} + i\frac{\sqrt{3}}{2}$$

$$z_4 : \text{kat} = \frac{\frac{2\pi}{3} + 4\pi}{4} = \frac{\pi}{6} + \pi = \frac{7\pi}{6}$$

$$\Rightarrow z_4 = \cos\left(\frac{7\pi}{6}\right) + i\sin\left(\frac{7\pi}{6}\right) = -\frac{\sqrt{3}}{2} - i\frac{1}{2}$$

$$z_5 : \text{kat} = \frac{\frac{2\pi}{3} + 6\pi}{4} = \frac{\pi}{6} + \frac{3\pi}{2} = \frac{5\pi}{3}$$

$$z_5 = \cos\left(-\frac{\pi}{3}\right) + i\sin\left(-\frac{\pi}{3}\right) = \frac{1}{2} - i\frac{\sqrt{3}}{2}$$

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④ $\left\{1+\frac{2+(-1)^n}{n} : n\in\mathbb{N}\right\} = A$

$n=1: \quad 1+\frac{1}{1} = 2$

$n=2: \quad 1+\frac{3}{2} = \frac{5}{2}$

$n=3: \quad 1+\frac{1}{3} = \frac{4}{3}$

$n=4: \quad 1+\frac{3}{4} = \frac{7}{4}$

$\max A = \frac{5}{2}$

$\frac{5}{2}$ je v množici ✓

in vsi elementi so manjši

$$1+\frac{2+(-1)^n}{n} \leq \frac{5}{2}$$

łatwo: $n = \text{scd} = 2k, \quad k = 1, 2, \dots$

$$1 + \frac{2+1}{2k} \le \frac{5}{2}$$

$$\frac{\cancel{3}}{2k} \le \frac{\cancel{3}}{2} \quad / \cdot 2k$$

$$1 \le k$$

$n = \text{lelu} = 2k+1, \quad k = 0, 1, \dots$

$$1 + \frac{2-1}{2k+1} \le \frac{5}{2}$$

$$\frac{1}{2k+1} \le \frac{3}{2} \quad / \cdot 2(2k+1)$$

$$2 \le 6k+3$$

$$-1 \le 6k$$

$$-\frac{1}{6} \le k$$

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$$\Rightarrow \max A = \frac{5}{2}$$

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$$\sup A = \frac{5}{2}$$

opazimo, da členi padajo proti 1.

$\inf A = 1$

a) 1 je spodnja meja (5)

$$1 + \frac{2+(-1)^n}{n} > 1$$

$$\frac{2+(-1)^n}{n} > 0 \quad / \cdot n$$

$$2+(-1)^n > 0 \quad \checkmark$$

1 ali 3

b) 1 je natančna spodnja meja (5)

Naj bo $\alpha > 1$. Iščemo n_0, da bo

$$1 + \frac{2+(-1)^{n_0}}{n_0} < \alpha$$

oziroma

$$2 + (-1)^{n_0} < n_0 (\underbrace{\alpha - 1}_{> 0})$$

$$n_0 > \frac{2 + (-1)^{n_0}}{\alpha - 1}.$$

Iščemo sod indeks na primer.

$$n_0 = 2k_0$$

$$\Rightarrow 2k_0 > \frac{3}{\alpha - 1} \Rightarrow k_0 > \frac{\frac{3}{2}}{\alpha - 1}$$

tak k_0 pa obstaja

$\Rightarrow 1$ je natančna spodnja meja

$$\Rightarrow \inf A = 1$$

limita A ne obstaja

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ker $1 + \frac{2+(-1)^n}{n} = 1$ ni res

odvisno za

noben n

$\frac{2+(-1)^n}{n} = 0 \quad / n$

$2+(-1)^n = 0$ ✗

$\underbrace{2+(-1)^n}$

1 ali 3

$\Rightarrow$ limita A ne obstaja