Preslikave

1. Naj bo $F\colon \mathbb{R}^2 \to \mathbb{R}^3$ podana s predpisom

$$f(x,y) = \left((x+y^2)^3, \sin(xy^2), \ln(1+x^2+y^2)\right).$$

   Izračunaj $JF(0,1)$.

   Rešitev: $JF(0,1) = \begin{bmatrix} 3 & 6 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$

2. Naj bo $F(x) = (x^5, \tan x^2, \arctan x^2)$. Določi definicijsko območje in izračunaj Jacobijevo matriko.

   Rešitev: $\mathcal{D}_F = \mathbb{R} \backslash \left\{\pm\sqrt{\frac{\pi}{2} + k\pi} : k \in \mathbb{N}_0\right\}$

   $JF = \begin{bmatrix} 5x^4 \\ \frac{2x}{\cos^2 x^2} \\ \frac{2x}{1+x^4} \end{bmatrix}$

3. Spremenljivki u in v sta diferenciabilni funkciji x, y in z, le-te pa so nadaljnje diferenciabilne funkcije spremenljivk t in s. V neki točki velja:

$$\frac{\partial u}{\partial x} = 2, \quad \frac{\partial u}{\partial y} = 3, \quad \frac{\partial u}{\partial z} = 1,$$

$$\frac{\partial v}{\partial x} = -1, \quad \frac{\partial v}{\partial y} = 0, \quad \frac{\partial v}{\partial z} = 3,$$

$$\frac{\partial x}{\partial s} = 5, \quad \frac{\partial y}{\partial s} = -6, \quad \frac{\partial z}{\partial s} = 4,$$

$$\frac{\partial x}{\partial t} = -3, \quad \frac{\partial y}{\partial t} = 2, \quad \frac{\partial t}{\partial s} = 3.$$

   Spremenljivki u in v lahko gledamo tudi kot funkciji spremenljivk s in t. V dani točki izračunajte parcialne odvode

$$\frac{\partial u}{\partial s}, \quad \frac{\partial u}{\partial t}, \quad \frac{\partial v}{\partial s}, \quad \frac{\partial v}{\partial t}.$$

Rešitev:

$$\frac{\partial u}{\partial s} = -4, \quad \frac{\partial u}{\partial t} = 3, \quad \frac{\partial v}{\partial s} = 7, \quad \frac{\partial v}{\partial t} = 12.$$

4. Spremenljivke u, v in w so diferenciabilne funkcije x in y, le-ti pa se izražata v polarnih koordinatah: $x = r\cos\varphi$, $y = r\sin\varphi$. Pri $x = 3$ in $y = 4$ velja:

$$\frac{\partial u}{\partial x} = 2, \quad \frac{\partial u}{\partial y} = -1,$$

$$\frac{\partial v}{\partial x} = 3, \quad \frac{\partial v}{\partial y} = 0,$$

$$\frac{\partial w}{\partial x} = 1, \quad \frac{\partial w}{\partial y} = 2.$$

Spremenljivki u, v in w lahko gledamo tudi kot funkcije spremenljivk r in φ. V dani točki izračunajte vse možne parcialne odvode prvega reda spremenljivk u, v in w po r in φ.

Rešitev:

$$\frac{\partial u}{\partial r} = \frac{2}{5}, \quad \frac{\partial u}{\partial \varphi} = -11, \quad \frac{\partial v}{\partial r} = \frac{9}{5}, \quad \frac{\partial v}{\partial \varphi} = -12, \quad \frac{\partial w}{\partial r} = \frac{11}{5}, \quad \frac{\partial w}{\partial \varphi} = 2.$$