Vectors

1. Leaving the outpost, Alejandro went on a three-day journey into the desert. His displacement (distance and direction) from start to finish on day one was $u = (3, 7)$. His displacement from start to finish on day two was $v = (-1, 8)$. (All distances are given in km.) His displacement on day three, w, took him back to the outpost. What distance did Alejandro travel on day three?

2. a) Sketch $u = (4, 0, 0)$ and $v = (0, 3, 0)$.
   b) Calculate the cross product of u and v. Add this vector to your sketch.
   c) Based on your sketch, what seems to be the angle between u and $u \times v$?
   d) Is there a way to verify this mathematically?
   e) When this is the case, we say that u is __________________ to v.
   f) Calculate the cross product of u and the vector $2u$.
   g) What is the length of the vector $u + v$?
   h) What is the length of $u \times v$?
   i) Calculate the length of $(u + v) \times u$. (What does this quantity correspond to geometrically?)
   j) Let $w = (1, 1, 3)$. Calculate the length of the vector $u \cdot (v \times w)$? (What does this quantity correspond to geometrically?)

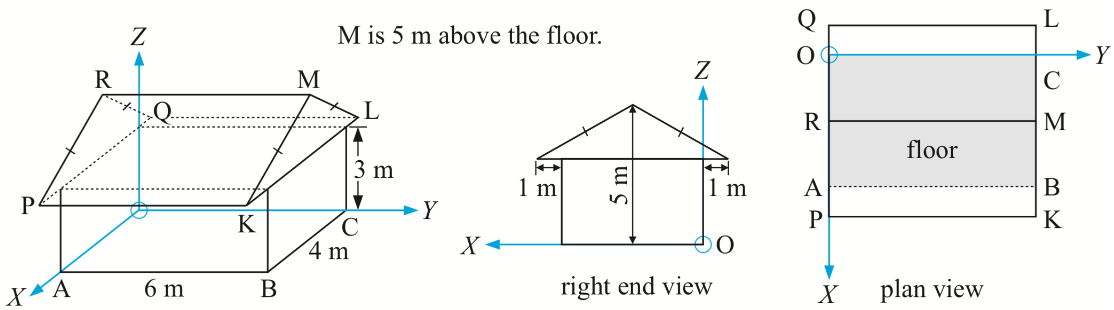
3. In $\mathbb{R}^3$ we are given three points $A(5, -2, 2)$, $B(3, -4, 6)$ and $C(2, 1, -1)$.
   a) Compute the length of the line segment AB.
   b) Compute the angle $\angle BAC$.

4. Let $\vec{a} = (0, 1, 2)$ and $\vec{b} = (1, 2, 3)$. Find x and y such that the vector $\vec{c} = (1, x, y)$ will be orthogonal to $\vec{a}$ and $\vec{b}$.

5. A garage is illustrated below. Three-dimensional coordinate axes have been inserted for convenience. Three-dimensional geometry enables us to solve problems in space provided that each point can be specified by coordinates.
   a) Can you specify the coordinates of the midpoint of RM?
   b) Can you find the distance PC?

c) Can you find (the coordinates of) a point on AC that divides AC in the ratio 2:3?

d) Can you write the vector for QK?



6. Vectors $\vec{v}_1$, $\vec{v}_2, \ldots, \vec{v}_k$ are said to be *linearly dependent* if there exist scalars $a_1, a_2, \ldots, a_k$ not all zero, such that

$$a_1\vec{v}_1 + a_2\vec{v}_2 + \ldots + a_k\vec{v}_k = 0,$$

i.e. one vector is zero or a linear combination of the others. Vectors $\vec{v}_1$, $\vec{v}_2, \ldots, \vec{v}_k$ are said to be *linearly independent* if the equation

$$a_1\vec{v}_1 + a_2\vec{v}_2 + \ldots + a_k\vec{v}_k = 0$$

can only be satisfied by $a_i = 0$ for all $i = 1, \ldots, k$.

a) Show that $\vec{v}_1 = (2, 5)$ and $\vec{v}_2 = (-1, 2)$ are linearly independent.

7. In trapezoid $ABCD$ the sides AB and CD are parallel. In what ratio does the diagonal AC divide the diagonal BD if $|AB| = 3|CD|$?