Rešitve

1. kolokvija

MAT II

NALOGA 1

a) $f(x,y)$ zvezna v $(0,0)$

polarne koordinate: $x = r\cos\varphi$

$y = r\sin\varphi$

$$\lim_{r\to 0} \frac{(r^2\cos^2\varphi - r^2\sin^2\varphi)^2}{r^2} =$$

$$\lim_{r\to 0} \frac{r^4\cos^2 2\varphi}{r^2} = \lim_{r\to 0} r^2\cos^2 2\varphi = \underline{\underline{0}}$$

$$\Rightarrow \boxed{a = 0}$$

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b) diferenciabilnost v $(0,0)$

$$f_x(0,0) = \lim_{h\to 0} \frac{f(h,0) - f(0,0)}{h}$$

$$= \lim_{h\to 0} \frac{h^4}{h^3} = \lim_{h\to 0} h = \underline{\underline{0}}$$

$$f_y(0,0) = \lim_{k \to 0} \frac{f(0,k) - f(0,0)}{k}$$

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$$= \lim_{k \to 0} \frac{k^4}{k^2 \cdot k} = \lim_{k \to 0} k = \underline{\underline{0}}$$

ogledamo si limito

$$\lim_{k,h \to 0} \frac{f(h,k) - f(0,0) - f_x(0,0)h - f_y(0,0)k}{\sqrt{h^2+k^2}}$$

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$$= \lim_{k,h \to 0} \frac{(h^2-k^2)^2}{\sqrt{h^2+k^2} \cdot (h^2+k^2)}$$ polarne koordinate

$$= \lim_{r \to 0} \frac{r^4 \cos^2 2\varphi}{r \cdot r^2} = \lim_{r \to 0} r \cos^2 2\varphi$$

$$= \underline{\underline{0}}$$

$\Rightarrow$ f je diferenciabilna v $(0,0)$

NALOGA 2

$$f(x,y) = x^3 + 12x^2 - 6xy - 21x + y^2 + 4y$$

a) lokalni ekstrem:

$$f_x = 3x^2 + 24x - 6y - 21 = 0$$

$$f_y = -6x + 2y + 4 = 0 \Rightarrow$$

$$2y = 6x - 4$$

$$y = 3x - 2$$

$$3x^2 + 24x - 18x + 12 - 21 = 0$$

$$3x^2 + 6x - 9 = 0$$

$$x^2 + 2x - 3 = 0$$

$$(x+3)(x-1) = 0$$

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$$x_1 = -3, \quad x_2 = 1$$

$$y_1 = -11 \qquad y_2 = 1$$

$$T_1(-3, -11) \qquad T_2(1, 1)$$

klasifikacija:

$$f_{xx} = 6x + 24$$

$$f_{xy} = -6$$

$$f_{yy} = 2$$

$T_1(-3,-11)$

$$\downarrow$$

$$H_1 = \begin{pmatrix} 6 & -6 \\ -6 & 2 \end{pmatrix}$$

$$\det H_1 = 12 - 36 < 0$$

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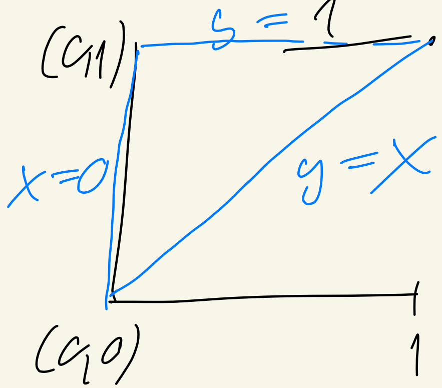
$\Rightarrow$ T_1 ni ekstrem

$T_2(1,1)$

$$H_2 = \begin{pmatrix} 30 & -6 \\ -6 & 2 \end{pmatrix} \Rightarrow \det H_2 = 60 - 36 = 24 > 0$$

ker je $f_{xx} > 0 \Rightarrow T(1,1)$ je lokalni minimum

5)



preveriva oglišča $A(0,1)$, $B(0,0)$, $C(1,1)$

$f(A) = 1 + 4 = 5$

$f(B) = 0$

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$f(C) = 1 + 12 - 6 - 21 + 1 + 4 = -9$

robovi:

$x = 0$

$g(y) = f(0,y) = y^2 + 4y$ ekstrem na $(0,1)$

$g'(y) = 2y + 4 = 0 \Rightarrow y = -2 \notin (0,1)$

② $y = 1$

$$g(x) = f(x,1) = x^3 + 12x^2 - 6x - 21x + 1 + 4$$

$$= x^3 + 12x^2 - 27x + 5$$

ekstrem na $[0,1]$

$$g'(x) = 3x^2 + 24x - 27 = 0$$

$$\Rightarrow x^2 + 8x - 9 = 0$$

$$(x+9)(x-1) = 0$$

$$x_1 = -9, \quad x_2 = 1 \Rightarrow T(1,1)$$

③ $y = x$

$$f(x,x) = x^3 + 12x^2 - 6x^2 - 21x + x^2 + 4x$$

$$= x^3 + 7x^2 - 17x$$

odvod $= 3x^2 + 14x - 17 = 0$

$$D = 14^2 + 4 \cdot 3 \cdot 17 = 196 + 4 \cdot 51$$
$$= 196 + 204$$
$$= 400$$

$$\Rightarrow x_{1,2} = \frac{-14 \pm 20}{6} \begin{cases} \nearrow 1 \ \checkmark \\ \searrow -\frac{34}{6} \ // \end{cases}$$

$\Rightarrow T(1,1)$ je globalni minimum z vrednostjo -9

$T(9,1)$ je globalni maksimum z vrednostjo 5

(10)

NACOBA 3

$$f(x,y) = \frac{\cos x}{1-y^2}$$

a) $T_6 = ?$

$$\cos y = 1 - \frac{y^2}{2} + \frac{y^4}{4!} - \frac{y^6}{6!}$$

$$\frac{1}{1-y^2} = 1 + y^2 + y^4 + y^6$$

$$= \frac{\cos x}{1-y^2} = 1 + y^2 + y^4 + y^6 - \frac{x^2}{2} - \frac{x^2y^2}{2} - \frac{x^2y^4}{2} + \frac{x^4}{4!} + \frac{x^4y^2}{4!} - \frac{x^6}{6!}$$

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5) $f_{xxxyyy}(0,0) = ?$

člen x^3y^3 ne nastopa v razvoju

$\Rightarrow f_{xxxyyy}(0,0) = 0$ 5

$f_{yyyyyy}(0,0) = ?$

primerjamo koeficiente

$$1 = \frac{1}{6!}\binom{6}{6}\frac{\partial^6 f}{\partial y^6}(0,0)$$

$$\Rightarrow \frac{\partial^6 f}{\partial y^6}(0,0) = 6!$$

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c) $T_6(0{,}1\,;\,-0.1)$

$$= 1 + \left(\frac{1}{10}\right)^2 + \left(\frac{1}{10}\right)^4 + \left(\frac{1}{10}\right)^6 - \frac{1}{2}\left(\frac{1}{10}\right)^2 - \frac{1}{2}\left(\frac{1}{10}\right)^4 - \frac{1}{2}\left(\frac{1}{10}\right)^6 + \frac{1}{4!}\left(\frac{1}{10}\right)^4 + \frac{1}{4!}\left(\frac{1}{10}\right)^6 - \frac{1}{6!}\left(\frac{1}{10}\right)^6$$

$$= 1 + \left(\frac{1}{10}\right)^2\left(1 - \frac{1}{2}\right) + \left(\frac{1}{10}\right)^4\left(1 - \frac{1}{2} + \frac{1}{4!}\right) + \left(\frac{1}{10}\right)^6\left(1 - \frac{1}{2} + \frac{1}{4!} - \frac{1}{6!}\right)$$

$$\doteq 1.0050547069$$

prava vrednost: ... 54712

NALOGA 4

$x=0,\ y=2$

$\cdot\ f(0,2) = 9-9 = \underline{\underline{0}}$ ✓

$F_x = 2(4x^2+y^2-4)\cdot 8x$

$\quad + 2(x^2+y^2-1)\cdot 2x$

$F_x(0,2) = 0$ ✓

$F_y = 2\cdot(4x^2+y^2-4)\cdot 2y$

$\quad +2(x^2+y^2-1)\cdot 2y$

$F_y(0,2) = 4\cdot 2\cdot 3 = 24 \neq 0$ ✓

Po izreku o implicitni funkcija obstaja okolica U točke $x=0$ in funkcija $g(x)$ da velja $g(0)=2$ in $F(x,g(x))=0$ ✓

b) $x=0$ je stacionarna točka za g.

Po I. I. P. velja $g'(0) = -\frac{F_x}{F_y}$

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$= 0$

$\Rightarrow x=0$ je stacionarna točka