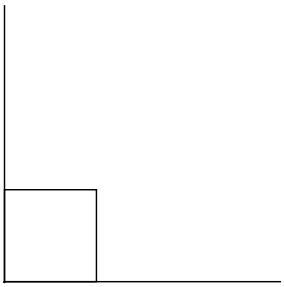
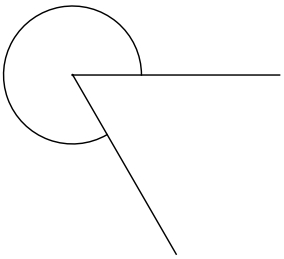
Expressing Mathematics in English - Midterm Test

10 February 2020

Name **Enrolment number** **Signature**

(1) Give appropriate names.





(2) Draw a square together with its inscribed circle.

(3) Draw a simple polygon and one that's not simple (and indicate which is which).

(4) Describe in words how to calculate the scalar product of two vectors.

(5) Name and describe two properties (axioms) of the integers.

(6) Complete the definitions below (in sentences).

(a) Let A be a planar set. Then A is called convex if

(b) A triangle is called isosceles if

(c) Let f be a real valued function defined on a suitable subset A of the real line. The function f is increasing if

(d) Let f be a real valued function defined on a suitable subset A of the real line, and let x_0 be a point in A. Let y be a real number. Then y is said to be the limit of f, as x tends to x_0, if

(7) Logically negate the sentences and express the negations as simply as possible.

(a) All dogs bark.

(b) If you haven't eaten fruit, I am not going to eat bread.

(c) For any integer n, the number $(3n-1)^2 - 1$ is divisible by three.

(8) Fill in the blanks with appropriate words.

The concept of an infinite series _____ the idea of _____ up infinitely many numbers listed in a given _____.
For an infinite series $a_1 + a_2 + a_3 + \ldots$, the quantity $s_n = a_1 + a_2 + \cdots + a_n$, which involves _____ only the first n _____, is called a partial sum of the series. If s_n _____ a fixed number S as n becomes _____ and _____, the series is said to _____. In this case, S is called the _____ of the series. An infinite series that does not _____ is said to _____. In the case of divergence, no value of a _____ is assigned. For example, the nth _____ _____ of the infinite series $1 + 1 + 1 + \ldots$ is n. As more _____ are added, the partial sums fail to _____ any finite value (they grow without _____). Thus, the series _____.
An example of a _____ series is

$$1 + \tfrac{1}{2} + \tfrac{1}{4} + \tfrac{1}{8} + \ldots$$

As n becomes _____, the nth partial sum _____ 2, which is the _____ of this infinite series. In fact, the series $1+q+q^2+q^3+\ldots$ (in the example above, q equals $1/2$) _____ to the sum $1/(1-q)$ if $-1 < q < 1$ and _____ otherwise. This series is called the _____ series with ratio q and was one of the first infinite series to be studied.

(9) Translate:
Vsak nekonstanten polinom (v eni spremenljivki) s kompleksnimi koeficienti ima vsaj eno ničlo v obsegu kompleksnih števil.

(10) Read the following text.

PARTIAL FRACTIONS

To write a sum or difference of fractional expressions as a single fraction, we bring them to a common denominator. For example,

$$\frac{1}{x-1}+\frac{1}{2x+1}=\frac{(2x+1)+(x-1)}{(x-1)(2x+1)}=\frac{3x}{2x^2-x-1}$$

But for some applications of algebra to calculus we must reverse this process–that is, we must express a fraction such a s $3x/(2x^2-x-1)$ as the sum of the simpler fractions $1/(x-1)$ and $1/(2x+1)$. These simpler fractions are called *partial fractions*; we learn how to find them in this section.

Let r be the rational function

$$r(x)=\frac{P(x)}{Q(x)}$$

where the degree of P is less than the degree of Q. By the Linear and Quadratic Factors Theorem, every polynomial with real coefficients can be factored completely into linear and irreducible quadratic factors, that is, factors of the form $ax+b$ and ax^2+bx+c where a, b, and c are real numbers. For instance,

$$x^4-1=(x^2-1)(x^2+1)=(x-1)(x+1)(x^2+1)$$

After we have completely factored the denominator Q of r, we can express $r(x)$ as a sum of **partial fractions** of the form

$$\frac{A}{(ax+b)^i} \quad \text{and} \quad \frac{Ax+B}{(ax^2+bx+c)^j}$$

This sum is called the **partial fraction decomposition** of r.

Decide whether the statements below are true (T) or false (F).

__ Decomposition into partial fractions deals with bringing several fractional expressions to a common denominator.

__ The polynomial $p(x)=x^4-1$ can be factored completely into factors of form a_ix+b_i where a_i and b_i are real numbers.

__ The method of decomposition into partial fractions applies to a rational function $r(x)=\frac{P(x)}{Q(x)}$ where the degree of the polynomial Q is more that the degree of the polynomial P.

__ For the method of decomposition into partial fractions we have to completely factor the numerator of the rational function in question.

__ The equality $\frac{3x}{2x^2-x-1}=\frac{x^2+1}{x-1}-\frac{x^2}{x-1}+\frac{1}{2x+1}$ is a decomposition into partial fractions.

__ Decomposition into partial fractions is about expressing a rational function as the sum of simple fractions.