6 Limite in zveznost

Naloga 6.1. Izračunajte naslednje limite funkcij.

(a) $\lim\limits_{x\to 0} \dfrac{x^3+6}{x+1}$

(b) $\lim\limits_{x\to -1} \dfrac{x^2-x-2}{x+1}$

(c) $\lim\limits_{x\to \infty} \dfrac{2x^2-3x}{x^2+2}$

(d) $\lim\limits_{x\to \infty} \dfrac{x^6+27}{x^7+16}$

(e) $\lim\limits_{x\to \infty} \dfrac{2x^5-7x}{5x^5-x^4+2x^3-6x^2+7x-2}$

(f) $\lim\limits_{x\to 0} \dfrac{\sin(2x)}{x}$

(g) $\lim\limits_{x\to \infty} x\sin(\frac{1}{x})$

(h) $\lim\limits_{x\to \frac{\pi}{2}} \dfrac{1-\sin(x)}{(\frac{\pi}{2}-x)^2}$

(i) $\lim\limits_{x\to \infty} \left(\dfrac{x}{x-1}\right)^x$

(j) $\lim\limits_{x\to \infty} \left(\dfrac{x^2-1}{x^2+1}\right)^{x^2+x+3}$

(k) $\lim\limits_{x\to 0} (1+3x)^{\frac{2}{\sin(x)}}$

(l) $\lim\limits_{x\to 1} \dfrac{\sqrt{x}-1}{\sqrt[3]{x}-1}$

Naloga 6.2. Za naslednji funkciji izračunajte levo in desno limito, ko gre x proti 0.

(a) $\arctan(\frac{1}{x})$

(b) $e^{\frac{1}{x}}$

Naloga 6.3. Ali so naslednje funkcije $f\colon \mathbb{R}\to\mathbb{R}$ zvezne?

(a) $f(x)=\begin{cases} \frac{\sin(\frac{x}{2})}{x}, & \text{če } x>0 \\ \frac{1}{2}-3x^2, & \text{če } x\le 0 \end{cases}$

(b) $f(x)=\begin{cases} \cos(x), & \text{če } |x|\le \frac{\pi}{2} \\ |x-\frac{\pi}{2}|, & \text{če } |x|>\frac{\pi}{2} \end{cases}$

(c) $f(x)=\begin{cases} \frac{x^2}{1-\sqrt{1-x^2}}, & \text{če } x<0 \\ 2-x, & \text{če } x\ge 0 \end{cases}$

Naloga 6.4. Določite k tako, da bo $f\colon \mathbb{R}\to\mathbb{R}$, $f(x)=\begin{cases} \frac{x^3+1}{x+1}, & \text{če } x\neq -1 \\ k, & \text{če } x=-1 \end{cases}$ zvezna.