Sequences and series

1. A) Try to determine the pattern in each of the following.

    a) $1, 2, 3, 4, 5, \ldots$

    b) $2, 4, 8, 16, 32, 64, 128, 256, 512, 1024, 2048, 4096, 8192 \ldots$

    c) $1, 3, 7, 15, 31, 63, 127, 255, 511, 1023, 2047, 4095, 8191 \ldots$

    d) $3, 7, 31, 127, 2047, 8191, \ldots$

    e) $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5} \ldots$

    f) $\frac{1}{2}, \frac{1}{6}, \frac{5}{12}, \frac{13}{60}, \frac{23}{60} \ldots$

    g) 0,1,0,1,0,1,0,1,...

    B) Which of the above sequences, if any, do you think converge?

2. Recall that every bounded monotone sequence in $\mathbb{R}$ is convergent. We can use this to test whether or not a given sequence converges. For example, consider the sequence $(1 + \frac{1}{p_i})_{i \geq 1}$ where p_i represents the i-th prime number:

$$\frac{3}{2}, \frac{4}{3}, \frac{6}{5}, \frac{8}{7}, \frac{12}{11}, \frac{14}{13}, \frac{18}{17}, \frac{20}{19}, \ldots$$

    Does this converge?

3. What are some adjectives that can be used to describe sequences?

4. Which of the following could be the first 5 terms of a convergent sequence?

    a) $1.1, 1.01, 1.001, 1.0001, 1.00001$

    b) $1, 2, 3, 4, 5$

    c) $10, 10^8, 10^{16}, 10^{32}, 10^{100}$

    d) $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}$

    e) $\frac{1}{2}, \frac{1}{6}, \frac{5}{12}, \frac{23}{60}$

5. Do you know what these mathematical abbreviations stand for?

    a) iff

    b) TFAE

    c) WLOG

    d) wrt

    e) e.g.

    f) i.e.

6. Where is the mistake in the following 'proof' that $1 = 0$?

$$
\begin{aligned}
0 &= (1-1) + (1-1) + (1-1) + \ldots \\
&= 1 - 1 + 1 - 1 + 1 - 1 + \ldots \\
&= 1 + (-1+1) + (-1+1) + (-1+1) + \ldots \\
&= 1
\end{aligned}
$$

7. Give an example of each of the following or show that it does not exist.

    a) A sequence not containing 0 or 1 as a term but containing subsequences converging to each of these values.

    b) A monotone sequence that diverges but has a convergent subsequence.

    c) An unbounded sequence with a convergent subsequence.

    d) Every bounded sequence has a convergent subsequence.

    e) Two sequences which both diverge but their sum converges.

8. For each of the following series determine if the series converges or diverges. If the series converges give its value.

    1. $\displaystyle\sum_{n=1}^{\infty} \frac{5}{6n}$

    2. $\displaystyle\sum_{n=1}^{\infty} \frac{5^{n+1}}{7^{n-2}}$

    3. $\displaystyle\sum_{n=1}^{\infty} 3^{2+n} 2^{1-3n}$

    4. $\displaystyle\sum_{n=1}^{\infty} \frac{3}{n^2+7n+12}$

9. Compute $0.999\ldots$ using geometric series.