Quantifiers, negation and limits

1. Complete the following statements (to statements which are different from those from the lecture). There are obviously many right answers.

    (a) For all ...

    (b) For every ...

    (c) For any ...

    (d) All ...

    (e) Every ...

    (f) Any ...

    (g) There exists a ...

    (h) There is some ...

    (i) Does there exist ...

    (j) Is there a ...

    (k) ... be a ...

    (l) Take any ...

2. Write down the following sentences in symbols. Which of them are true?

    (a) There is an integer whose square is 2.

    (b) All integers are divisible by 5.

    (c) Some integer is divisible by 7.

    (d) The square of any integer is nonnegative.

    (e) The product of two rational numbers is always rational.

    (f) For every integer x, there is an integer y such that $\frac{x}{y} = 10$.

    (g) No matter what integer you choose, there is always another integer that is larger.

3. Write down in English and using symbols.

    (a) A statement with precisely one quantifier.

    (b) A statement with one existential quantifier and one universal quantifier.

4. Negate the following statements.

    (a) You are either rich or happy.

    (b) I am both rich and happy.

    (c) If n is even, then $\frac{n}{2}$ is an integer.

    (d) All Slovenes ride bicycles to work.

    (e) It will snow today or tomorrow.

5. In number theory, Fermat's Last Theorem states that no three positive integers a, b, and c satisfy the equation $a^n + b^n = c^n$ for any integer value of n greater than 2.

    (a) Write out Fermat's last theorem in symbols.

    (b) Negate this statement.

    (c) Write the negated statement out in words.

6. Let $f : \mathbb{R} \to \mathbb{R}$ be a real function. Let a be a real number. Consider the statement

$$\forall \varepsilon > 0 \, \exists \delta > 0 : \, |x - a| < \delta \Rightarrow |f(x) - L| < \varepsilon.$$

    (a) Write down this statement in words.

    (b) What is this statement telling us?

    (c) Write down the negation of this statement both using symbols and in words.

7. Compute the following limits:

    1. $\lim\limits_{x \to 3} \frac{\sqrt{x^2 - 5} - 2}{x - 3}$

    2. $\lim\limits_{x \to 2} \frac{x^3 - 8}{x^2 - 4}$

    3. $\lim\limits_{x \to 0} \frac{\ln(1+x)}{x}$

8. Let $f : \mathbb{R} \to \mathbb{R}$ be a real function. Consider the statement

$$\forall \varepsilon > 0 \, \exists x_0 \in \mathbb{R} \, \forall x \geq x_0 : \, |f(x)| < \varepsilon.$$

    (a) Write down this statement in words.

    (b) What is this statement telling us?

    (c) Write down the negation of this statement both using symbols and in words.

9. Compute the following limits:

    1. $\lim\limits_{x \to \infty} \frac{8 - 4x^2}{9x^2 + 5x}$

    2. $\lim\limits_{x \to \infty} \frac{x + 8}{\sqrt{2x^2 + 3}}$

    3. $\lim\limits_{x \to \infty} \left(\frac{x^3 + 1}{x^3 - 1} \right)^{x^3 + 2x}$