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DISCRETE MATHEMATICS 2

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5. Ramsey theory

Exercises.

5.1 18 teams participate at a round-robin soccer tournament. Prove that after eight rounds are played, we can still find three teams no two of which have played each other yet.

5.2 Each point of the space is colored either red or blue. Prove that either there is a unit square whose vertices are all blue, or there is a unit square that has at least three red vertices.

5.3 Prove that every edge-colouring of $\mathcal{K}_6$ with 2 colours has at least 2 monochromatic triangles (not necessarily of the same colour). Give a colouring of $\mathcal{K}_6$ with exactly two monochromatic triangles.

5.4 Prove that there exists a natural number $R(k_1, \ldots, k_s)$ such that if $n \geqslant R(k_1, \ldots, k_s)$ and we colour the edges of $\mathcal{K}_n$ with s colours, then there exists $i \in \{1, \ldots, s\}$ and a set V of k_i vertices that satisfis that all the edges of the complete graph induced by V are of colour i.

5.5 Prove that

$$R(\underbrace{3, \ldots, 3}_{s \text{ times}}) \leqslant 1 + s! \left(1 + \frac{1}{1!} + \frac{1}{2!} + \cdots + \frac{1}{k!}\right)$$

5.6 Let $n \geqslant 2$. Prove that $R(n+2, 3) > 3n$.

5.7 Prove that $R(3, 5) = 14$.

5.8 Prove that $R(4, 4) = 18$.

5.9 We colored each point of the space either red, or blue, or green or yellow. Prove that there is a segment of unit length with monochromatic vertices.

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5.10 Prove that it is possible to colour each point of the plane either red or blue so that there is no regular triangle with sides of unit length and monochromatic vertices.

5.11 We coloured each point of the plane either red or blue. Let T be any right-angled triangle. Prove that there is a triangle that is congruent to T and has monochromatic vertices.

5.12 A company has 2002 employees, from 6 different countries. Each employee has a company identification card (ID) with a number from 1 to 2002. Prove that there is either an employee whose ID number is equal to the sum of the ID numbers of two of his compatriots, or there is an employee whose ID number is twice that on one of compatriots.

5.13 Let us colour each positive integer by one of the colors $c_1, \ldots, c_k$.
- Prove that there exists an integer $N(k)$ so that if $n > N$, then there are three integer a, b, c that are less than n, are of the same colour and satisfy $a + b = c$ ($a = b$ is allowed).
- Determine $N(2)$.
- Prove that $N(3) > 13$.

5.14 There are 9 participants in a convention. None of the participants speak more than 3 languages. It is also true that 2 of each 3 participants speak a common language. Show that there are 3 participants that speak a common language.