MATEMATIKA 1 – 1. izpit

NALOGA 1

$$x^2 - 2|x+3| - 2 > 0$$

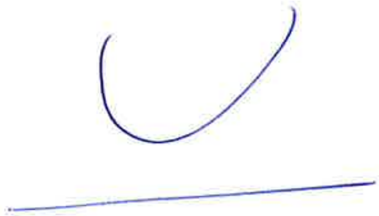
1.) $x < -3$

$$x^2 - 2(-1)(x+3) - 2 > 0$$

$$x^2 + 2x + 6 - 2 > 0$$

$$x^2 + 2x + 4 > 0$$

$D = 4 - 16 = -12 < 0 \Rightarrow$



vsak $x \in \mathbb{R}$ reši neenačbo

$\Rightarrow \underline{x < -3}$

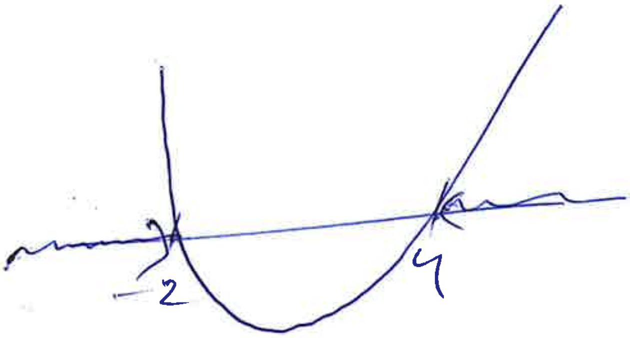
2.) $x \geq -3$

$$x^2 - 2x - 6 - 2 > 0$$

$$x^2 - 2x - 8 > 0$$

$$(x-4)(x+2) > 0$$

$x_1 = 4, \; x_2 = -2$



$\Rightarrow x \in [-3, -2) \cup (4, \infty)$

$\Rightarrow$ $\boxed{\text{rešitev: } x \in (-\infty, -2) \cup (4, \infty)}$

NALOGA 2

a) $$\lim_{n\to\infty} n\left(\sqrt{n^2+2}-\sqrt{n^2-3}\right)$$

$$= \lim_{n\to\infty} \frac{n\left(\sqrt{n^2+2}-\sqrt{n^2-3}\right)\left(\sqrt{n^2+2}+\sqrt{n^2-3}\right)}{\sqrt{n^2+2}+\sqrt{n^2-3}}$$

$$= \lim_{n\to\infty} \frac{n\left(\cancel{n^2}+2-\cancel{n^2}+3\right)}{\sqrt{n^2+2}+\sqrt{n^2-3}} = \lim_{n\to\infty} \frac{5n}{\sqrt{n^2+2}+\sqrt{n^2-3}} \begin{matrix} /:n \\ /:n \end{matrix}$$

$$= \lim_{n\to\infty} \frac{5}{\sqrt{1+\frac{2}{n^2}}+\sqrt{1-\frac{3}{n^2}}} = \underline{\underline{\frac{5}{2}}}$$

b) $$\lim_{x\to 0} \frac{xe^x-\ln(1+x)}{\sqrt{1+x^2}-1}$$

1. možnost: Taylor

$$xe^x = x\left(1+x+\frac{x^2}{2}+\dots\right) = x+x^2+\frac{x^3}{2}+\dots$$

$$\ln(1+x) = \ln(1-(-x)) = -\sum_{n=1}^{\infty}\frac{(-x)^n}{n}$$

$$= x-\frac{x^2}{2}+\frac{x^3}{3}+\dots$$

$$\sqrt{1+x^2} = (1+x^2)^{\frac{1}{2}} = \sum_{n=0}^{\infty}\binom{\frac{1}{2}}{n}x^{2n} = 1+\frac{1}{2}x^2+\dots$$

dosiwo

$$\lim_{x\to 0} \frac{x+x^2+\frac{x^3}{2}+\ldots - x+\frac{x^2}{2}-\frac{x^3}{3}+\ldots}{1+\frac{1}{2}x^2+\ldots \quad -1}$$

$$= \lim_{x\to 0} \frac{\frac{3x^2}{2}+o(x^3)}{\frac{1}{2}x^2+o(x^3)} = \lim_{x\to 0} \frac{\frac{3}{2}+o(x)}{\frac{1}{2}+o(x)} = 3$$

2. moivost : L'hopital

$$\lim_{x\to 0} \frac{e^x+xe^x-\frac{1}{1+x}}{\frac{1}{2}(1+x^2)^{-\frac{1}{2}}2x} =$$

$$\lim_{x\to 0} \frac{e^x+e^x+xe^x+\frac{1}{(1+x)^2}}{-\frac{1}{2}(1+x^2)^{-\frac{3}{2}}\cdot 2x\cdot x+(1+x^2)^{-\frac{1}{2}}} = 3$$

NALOGA 3

$$f(x) = \frac{2x+4}{\sqrt{x^2+\cancel{3}2}}$$

$DA = \mathbb{R}$

ničle: $2x+4=0 \Rightarrow \underline{x=-2}$

vodoravni asimptoti:

$$\lim_{x\to\infty} \frac{2x+4}{\sqrt{x^2+2}} = \lim_{x\to\infty} \frac{\cancel{3}2+\frac{\cancel{5}4}{x}}{\sqrt{1+\frac{2}{x^2}}} = \underline{2}$$

$$\lim_{x\to-\infty} \frac{2x+4}{\sqrt{x^2+\cancel{4}2}} = \lim_{x\to\infty} \frac{-2x+4}{\sqrt{x^2+2}} = \underline{-2}$$

ekstremi

$$f'(x) = \frac{2\sqrt{x^2+2} - (2x+4)\frac{1}{2}(x^2+2)^{-\frac{1}{2}} \cdot 2x}{(x^2+2)} = 0$$

$$\Rightarrow 2\sqrt{x^2+2} - (2x+4)x\frac{1}{\sqrt{x^2+2}} = 0 \quad / \sqrt{x^2+2}$$

$$2(x^2+2) - 2x^2 - 4x = 0$$

$$2x^2+4-2x^2-4x = 0$$

$$4\cancel{2}-4x = 0$$

$$x = 1$$

$$\Rightarrow f'(x) = \frac{\frac{1}{\sqrt{x^2+2}}(4-4x)}{(x^2+2)} = \frac{4-4x}{(x^2+2)^{\frac{3}{2}}}$$

$$f''(x) = \frac{-4(x^2+2)^{\frac{3}{2}} - (4-4x)\frac{3}{2}\cdot(x^2+2)^{\frac{1}{2}}\, 2x}{(x^2+2)^3}$$

$$= \frac{\sqrt{x^2+2}\,\left[-4(x^2+2) - 3x(4-4x)\right]}{(x^2+2)^3}$$

$$f''(1) = \frac{\sqrt{3}\,(-12)}{3^3} < 0 \Rightarrow x=1 \text{ je lokalni maksimum}$$

naraščaje / padaje:

$$f'(x) = \frac{4-4x}{(x^2+2)^{\frac{3}{2}}} > 0 \Leftrightarrow 4-4x > 0$$

$$4x < 4$$

$$\underbrace{x < 1}_{\text{narašča}}$$

$\Rightarrow$ pada za $x > 1$

pregib: $f''(x)=0$

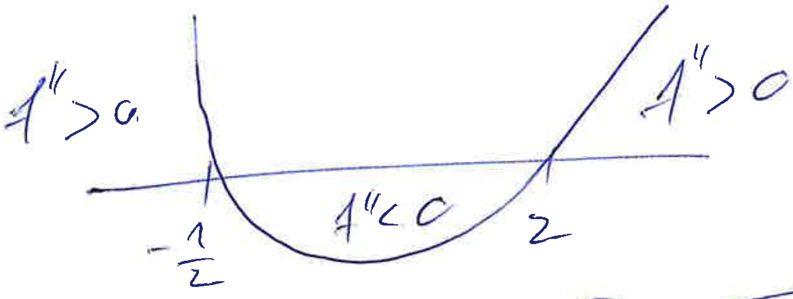
$\Rightarrow -4(x^2+2)-3x(4-4x)=0$

$-4x^2-8-12x+12x^2=0$

$8x^2-12x-8=0 \quad /:4$

$2x^2-3x-2=0$

$D=9+16=25 \Rightarrow x_{1,2}=\frac{3\pm 5}{4} \nearrow x_1=2, \ \searrow x_2=-\frac{1}{2}$

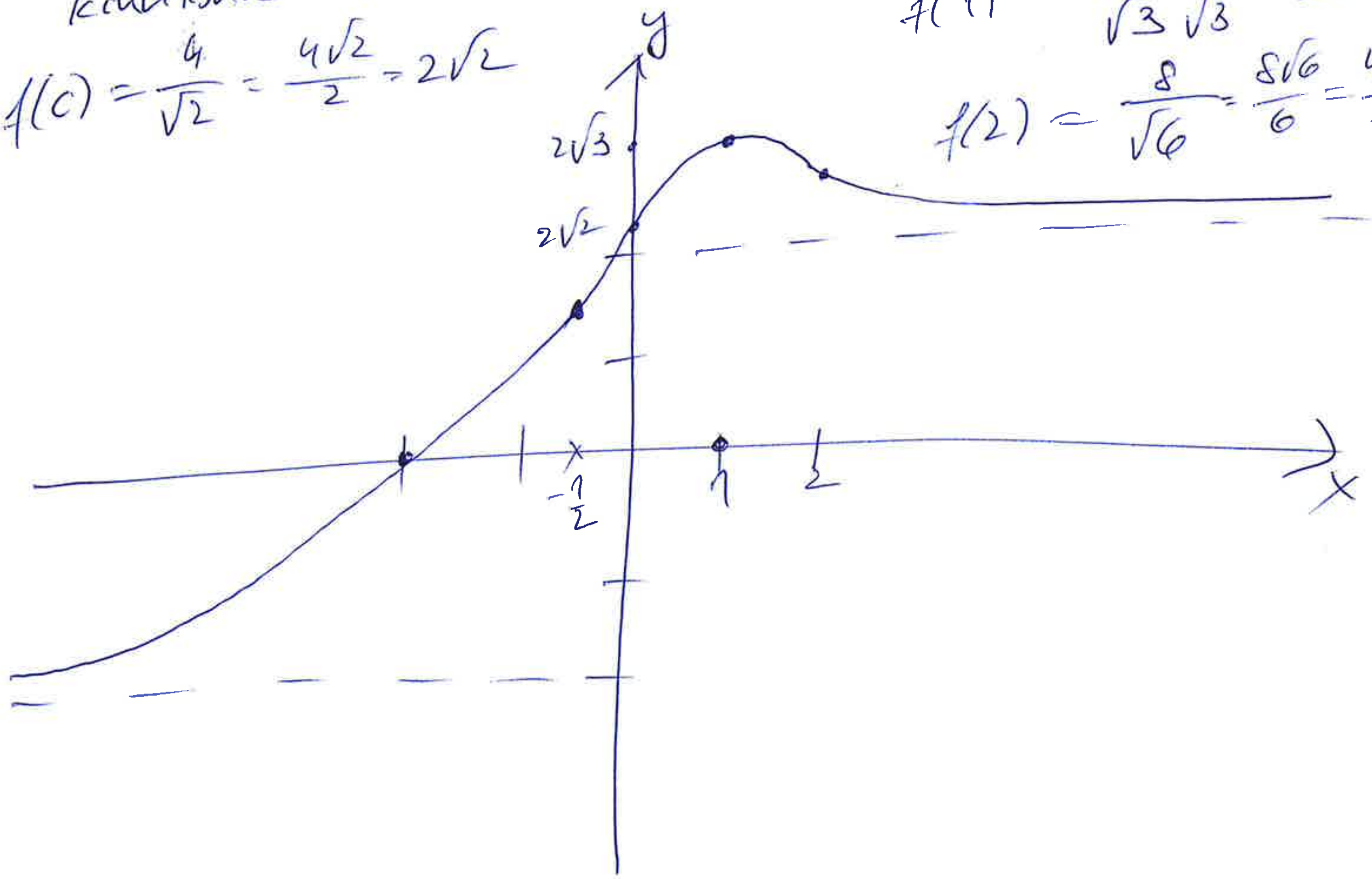


konveksna konkava konveksna

$f(1)=\frac{6\sqrt{3}}{\sqrt{3}\sqrt{3}}=2\sqrt{3}$

$f(0)=\frac{4}{\sqrt{2}}=\frac{4\sqrt{2}}{2}=2\sqrt{2}$

$f(2)=\frac{8}{\sqrt{6}}=\frac{8\sqrt{6}}{6}=\frac{4\sqrt{6}}{3}$



NALOGA 4

a) $\int x \arctan(x^2)\,dx = I$

1. način:

$u = \arctan(x^2)$ $\qquad dv = x\,dx$

$du = \frac{1}{1+x^4} \cdot 2x$ $\qquad v = \frac{x^2}{2}$

$\Rightarrow I = \frac{x^2 \arctan(x^2)}{2} - \int \frac{x^3}{1+x^4}\,dx$

$t = 1+x^4$

$dt = 4x^3\,dx$

$= \frac{x^2 \arctan(x^2)}{2} - \frac{1}{4}\ln(1+x^4) + C$

2. način:

$t = x^2,\ dt = 2x\,dx$

$\Rightarrow I = \frac{1}{2}\int \arctan t\,dt$

$u = \arctan t$ $\qquad dv = dt$

$du = \frac{1}{1+t^2}\,dt$ $\qquad v = t$

$\Rightarrow \frac{1}{2}\left(t \arctan t - \int \frac{t}{1+t^2}\,dt\right)$

$= \frac{1}{2}\left(t \arctan t - \frac{1}{2}\ln(1+t^2)\right) + C$

$$= \frac{1}{2}x^2 \operatorname{arctan} x^2 - \frac{1}{4}\ln(1+x^4) + C$$

b) ničle: $f(x) = 0 = \sin x + \cos x$

$$\sin x = -\cos x \quad / \cos x$$

$$\tan x = -1$$

$$x = -\frac{\pi}{4} + k\pi \Rightarrow \text{dve zaporedni ničli:}$$

$$x_1 = -\frac{\pi}{4}$$

$$x_2 = \frac{3\pi}{4}$$

$$\Rightarrow \int_{-\frac{\pi}{4}}^{\frac{3\pi}{4}} (\sin x + \cos x)\,dx = \left[-\cos x + \sin x\right]_{-\frac{\pi}{4}}^{\frac{3\pi}{4}}$$

$$= -\cos\frac{3\pi}{4} + \sin\frac{3\pi}{4} - \left(-\cos\frac{\pi}{4} - \sin\frac{\pi}{4}\right)$$

$$= -\left(-\frac{\sqrt{2}}{2}\right) + \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} = 4\cdot\frac{\sqrt{2}}{2} = \underline{\underline{2\sqrt{2}}}$$