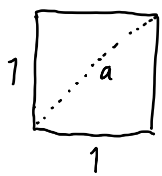
MANG L3

So far: Today:

$$\mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R}$$

Consider a square



what is the length of the diagonal of the unit square?

By Pythagoras' theorem

$$a^2 = 1^2 + 1^2 = 2$$

So $a = \sqrt{2}$ — the square root of 2

The ancient Greeks proved:

Theorem $\sqrt{2}$ is not a rational number

($\sqrt{2}$ is an irrational number)

Proof

Suppose for contradiction that $\sqrt{2}$ is rational.

such that

Then we have integers $m, n \geq 1$ s.t. $\sqrt{2} = \frac{m}{n}$

We can assume that m/n is in reduced form.

that is

We have $\frac{m^2}{n^2} = 2$; i.e., $m^2 = 2 \cdot n^2$.

So 2 divides m^2. Therefore 2 divides m.

We can write $m = 2m'$.

Because $m^2 = 2n^2$, we have $4(m')^2 = 2n^2$

that is $2(m')^2 = n^2$

So 2 divides n^2. Therefore 2 divides n.

Since 2 divides both m and n,

the fraction $\frac{m}{n}$ is not in reduced form.

This is a contradiction. Therefore $\sqrt{2}$ is irrational.

(as has been demonstrated) quod erat demonstrandum — Q.E.D.

How do we represent real numbers in general?

As decimal expansions

$$\sqrt{2} = 1.414\ 21356 \ldots$$

"One point four one four ..."

names $0, \ldots, 9$

We need infinitely many digits

There is no obvious pattern.

Ratios

Real numbers often arise as ratios.

Example the recipe for English porridge:

colon

1 part oats : 1 part milk : 1 part water

(quantities measured by volume)

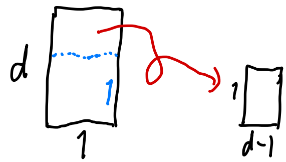
We say "one to one to one"

We can vary the relative proportion of milk and water, but the important <u>invariant</u> is the ratio:

1 part oats : 2 parts liquid

The golden ratio

The ancient Greeks considered that the most aesthetic rectangle is defined by the following ratio



the second rectangle is similar to the first

This means that

$$\frac{d}{1} = \frac{1}{d-1} \quad ; \text{ i.e., } d^2 - d - 1 = 0$$

This is a quadratic equation. It has the form $ax^2 + bx + c = 0$, with $a = 1$, $b = -1$, $c = -1$.

Remember the solutions are given by:

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$\phi \approx 1.6180339...$

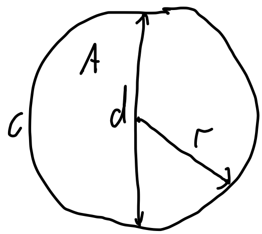
In our case we have

$$d = \frac{1 + \sqrt{1 + 4}}{2}$$

This gives the golden ratio

$$\phi = \frac{1 + \sqrt{5}}{2}$$

The most famous real number defined as a ratio.



$d =$ diameter

$C =$ circumference

$r =$ radius

$A =$ area

The ratio $c : d$ is the same irrespective of the size of the circle.

This defines the real number pi (don't confuse with pie)

$$\pi = \frac{c}{d}$$

π occurs ubiquitously in mathematics; eg,

By definition: $C = \pi d = 2\pi r$ circumference

$A = \pi r^2$ area of the circle

$V = \frac{4}{3}\pi r^3$ volume of a sphere

$4\pi r^2$ surface area of a sphere

Watch the video: " .. area of a circle"

- where is the speaker from?
- 2.58 "from thin air'
- 3.28 "concentric"
- 4.59 " unfurl "

what do these mean?

- At 6.12 what is the mistake in the subtitles?