(d) $(2x^2y^{-3}z^{-1})^{-3} : \left(\frac{2z^3x^2}{y^{-2}}\right)^4 = 2^{-3}x^{-6}y^9z^3 : \frac{2^4 z^{12} x^8}{y^{-8}} =$

$$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$$

$= 2^{-3} \cdot 2^{-4} x^{-6} \cdot x^{-8} y^9 y^{-8} z^3 \cdot z^{-12} =$

$$= \underline{\underline{2^{-7}x^{-14}yz^{-9}}}$$

(e) $\frac{x^2-y^2}{x^2-2xy+y^2} \cdot \frac{x^2-xy}{x^2+xy} = \frac{\cancel{(x-y)}\cancel{(x+y)}}{\cancel{(x-y)}^{\cancel{2}}} \cdot \frac{\cancel{x}\cancel{(x-y)}}{\cancel{x}\cancel{(x+y)}} = \underline{\underline{1}}$

$x^2-y^2 = (x-y)(x+y)$

$x^2 \pm 2xy + y^2 = (x \pm y)^2$

2. Izpostavi skupni faktor v naslednjem izrazu in izraz poenostavi.

$$2^{x+4} - 2^{x+3} - 3 \cdot 2^{x+2} + 5 \cdot 2^{x+1} =$$

$$= 2^{x+1}\left(2^3 - 2^2 - 3 \cdot 2 + 5\right) =$$

$$= 2^{x+1}\left(8 - 4 - 6 + 5\right) = 3 \cdot 2^{x+1}$$

3. Poenostavi naslednja izraza.

(a) $\frac{x^{n+1}-9x^{n-1}}{x^n}$

(b) $\frac{x^{n-1}}{x^n-2x^{n-1}}-\frac{2x^{n-1}}{x^n+2x^{n-1}}+\frac{x^n}{x^{n+1}-4x^{n-1}}$

(a) $\frac{\cancel{x^{n-1}}(x^2-9)}{\cancel{x^n}x} = \frac{x^2-9}{x}$ $\left(= \frac{(x-3)(x+3)}{x}\right)$

$9 = 3^2$

(b) $\frac{\cancel{x^{n-1}}}{\cancel{x^{n-1}}(x-2)} - \frac{2\cancel{x^{n-1}}}{\cancel{x^{n-1}}(x+2)} + \frac{x\cancel{x^{n-1}}}{\cancel{x^{n-1}}(x^2-4)} =$

$= \frac{1}{x-2} - \frac{2}{x+2} + \frac{x}{(x-2)(x+2)} = \frac{x+2-2(x-2)+x}{(x-2)(x+2)} =$

$= \frac{\cancel{x}+2-\cancel{2x}+4+\cancel{x}}{(x-2)(x+2)} = \frac{6}{x^2-4}$

4. Poenostavi naslednje izraze.

(a) $(\sqrt{3}+\sqrt{6})^2$

(b) $\sqrt{5+\sqrt{17}} \cdot \sqrt{5-\sqrt{17}}$

(c) $\sqrt[3]{a-b} \cdot \sqrt[3]{a^2-2ab+b^2}$

a) $(\sqrt{3}+\sqrt{6})^2 = 3 + 2\sqrt{3}\sqrt{6} + 6 = 9 + 2\sqrt{18} =$

$= 9 + 2 \cdot 3\sqrt{2} = \underline{\underline{9+6\sqrt{2}}}$

$18 = 9 \cdot 2$, $9 = 3^2$

$(a+b)^2 = a^2 + 2ab + b^2$

$\sqrt{a}\sqrt{b} = \sqrt{ab}$

$a^{\frac{1}{2}} \cdot b^{\frac{1}{2}} = (ab)^{\frac{1}{2}}$

(b) $\sqrt{5+\sqrt{17}}\cdot\sqrt{5-\sqrt{17}} = \sqrt{(5+\sqrt{17})(5-\sqrt{17})}$

$$= \sqrt{5^2-(\sqrt{17})^2} = \sqrt{25-17} = \sqrt{8} = \underline{\underline{2\sqrt{2}}}$$

(c) $\sqrt[3]{a-b}\cdot\sqrt[3]{a^2-2ab+b^2} =$

$$= \sqrt[3]{(a-b)(a-b)^2} = \sqrt[3]{(a-b)^3} = \underline{\underline{a-b}}$$

5. Racionaliziraj naslednja izraza.

(a) $\frac{1}{\sqrt{3}-1}$

(b) $\frac{3\sqrt{2}+2\sqrt{3}}{3\sqrt{2}-2\sqrt{3}}$

a) $$\frac{1}{\sqrt{3}-1} \cdot \frac{(\sqrt{3}+1)}{(\sqrt{3}+1)} = \frac{\sqrt{3}+1}{3-1} = \underline{\underline{\frac{\sqrt{3}+1}{2}}}$$

b) $$\frac{3\sqrt{2}+2\sqrt{3}}{3\sqrt{2}-2\sqrt{3}} \cdot \frac{3\sqrt{2}+2\sqrt{3}}{3\sqrt{2}+2\sqrt{3}} =$$

$$= \frac{18 + 2 \cdot 3\sqrt{2} \cdot 2\sqrt{3} + 12}{18 - 12} =$$

$$= \frac{18 + 12\sqrt{6} + 12}{6} = \frac{\cancel{6}(3 + 2\sqrt{6} + 2)}{\cancel{6}} = \underline{\underline{5 + 2\sqrt{6}}}$$

6. Poenostavi $\left(\sqrt[3]{xy^2}\right)^6 = \left(\left(xy^2\right)^{1/3}\right)^6 = \left(xy^2\right)^2 =$

$= \underline{\underline{x^2y^4}}$

7. Poenostavi

(a) $\sin(\alpha + 270°) - \cos(\alpha - 180°) =$

ADICIJSKA IZREKA

$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$
$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$

$= \cancel{\sin\alpha\cos 270°} + \cos\alpha\sin 270° -$
$- \cos\alpha\cos 180° - \cancel{\sin\alpha\sin 180°} =$

$= -\cos\alpha + \cos\alpha = \underline{\underline{0}}$

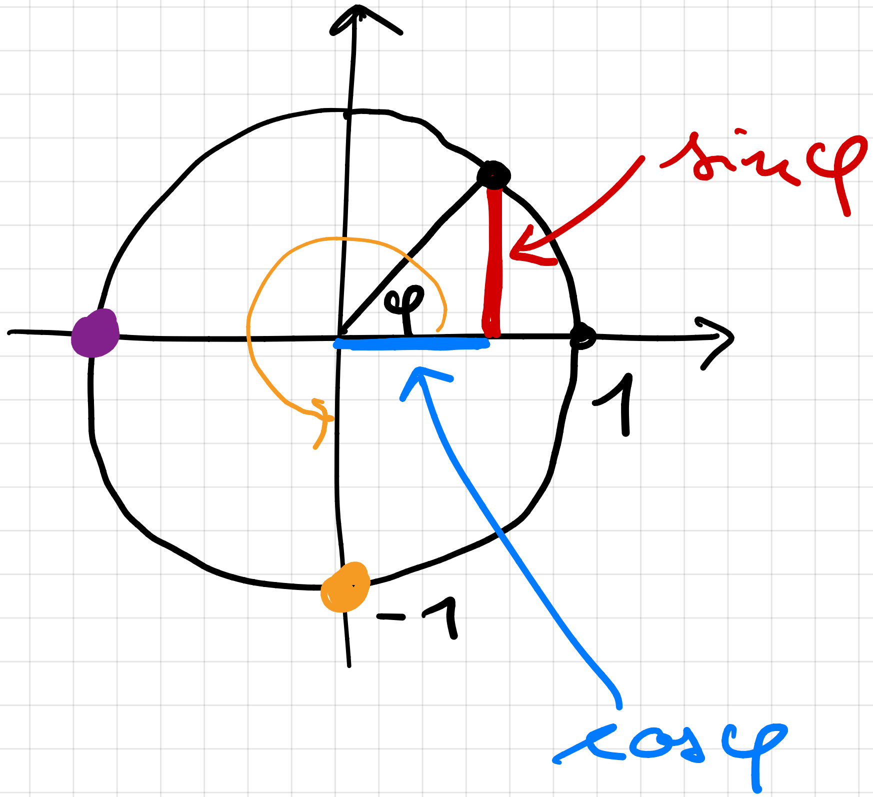
$$\cos 270^\circ = 0$$

$$\underline{180^\circ + 90^\circ}$$

$$\sin 270^\circ = -1$$

$$\cos \underline{180^\circ} = -1$$

$$\sin 180^\circ = 0$$



(c) $\sin(x - \frac{\pi}{6}) + \cos(x + \frac{\pi}{6}) =$

$$\left[\begin{array}{l} \sin(\alpha - \beta) = \sin\alpha\cos\beta - \sin\beta\cos\alpha \\ \cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta \end{array}\right]$$

$$= \sin x \cos\frac{\pi}{6} - \sin\frac{\pi}{6}\cos x + \cos x \cos\frac{\pi}{6} - \sin x \sin\frac{\pi}{6} =$$

$$= \sin x \frac{\sqrt{3}}{2} - \frac{1}{2}\cos x + \cos x \frac{\sqrt{3}}{2} - \sin x \cdot \frac{1}{2} =$$

$$= \frac{\sqrt{3}-1}{2}\sin x + \frac{\sqrt{3}-1}{2}\cos x =$$

$$= \frac{\sqrt{3}-1}{2}(\sin x + \cos x)$$

8. Poenostavi naslednje trigonometrijske izraze.

(a) $\cos\alpha \cdot \operatorname{tg}\alpha$,

$$\cos\alpha \cdot \operatorname{tg}\alpha = \cancel{\cos\alpha}\,\frac{\sin\alpha}{\cancel{\cos\alpha}} = \underline{\underline{\sin\alpha}}$$

(b) $(\sin\alpha - \cos\alpha)^2 + (\sin\alpha + \cos\alpha)^2$ =

$$= \sin^2\alpha - \cancel{2\sin\alpha\cos\alpha} + \cos^2\alpha +$$

$$+ \sin^2\alpha + \cancel{2\sin\alpha\cos\alpha} + \cos^2\alpha =$$

$$= 2\sin^2\alpha + 2\cos^2\alpha = 2(\underbrace{\sin^2\alpha + \cos^2\alpha}_{1}) = \underline{\underline{2}}$$

(c) $\left(\operatorname{tg}\alpha + \frac{1}{\cos\alpha}\right)\left(\operatorname{tg}\alpha - \frac{1}{\cos\alpha}\right) = \left(\frac{\sin\alpha}{\cos\alpha} + \frac{1}{\cos\alpha}\right) \cdot$

$$\cdot \left(\frac{\sin\alpha}{\cos\alpha} - \frac{1}{\cos\alpha}\right) =$$

$$= \frac{\sin\alpha + 1}{\cos\alpha} \cdot \frac{\sin\alpha - 1}{\cos\alpha} =$$

$$= \frac{\sin^2\alpha - 1}{\cos^2\alpha} =$$

$$= \frac{-\cancel{\cos^2\alpha}}{\cancel{\cos^2\alpha}} = \underline{\underline{-1}}$$

$$\cos^2\alpha + \sin^2\alpha = 1 \implies \sin^2\alpha - 1 = -\cos^2\alpha$$

(d) $\cos x \cdot (1+\operatorname{tg}^2 x) = \cos x \left(1 + \frac{\sin^2 x}{\cos^2 x}\right) =$

$$= \cos x \left(\frac{\cos^2 x + \sin^2 x}{\cos^2 x}\right) =$$

$$= \cancel{\cos x} \cdot \frac{1}{\cos^{\cancel{2}} x} = \frac{1}{\cos x}$$

MIMOGREDE:

$$\boxed{1 + \operatorname{tg}^2 x = \frac{1}{\cos^2 x}}$$

(e) $\sin 2x \cdot \operatorname{tg} x + 2\cos^2 x$ = I

$$\sin 2x = 2\sin x \cos x$$

$$I = 2\sin x \cancel{\cos x} \cdot \frac{\sin x}{\cancel{\cos x}} + 2\cos^2 x =$$

$$= 2\sin^2 x + 2\cos^2 x = 2(\underbrace{\sin^2 x + \cos^2 x}_{1}) = \underline{\underline{2}}$$

11. Reši naslednje enačbe.

(a) $x^2 - 2x + 1 = 0$

(b) $x^2 - 3x + 2 = 0$

(c) $x^3 - x^2 - x + 1 = 0$

(d) $x^3 - 8x = 0$

(e) $x^3 - 6x^2 + 6x - 1 = 0$

a) $x^2 - 2x + 1 = 0$

$(x-1)^2 = 0$

$k = 2$

$x = 1$ dvojna ničla

$$\boxed{\text{a je k-kratna ničla polinoma $p(x)$, če } p(x) = (x-a)^k q(x),\ q(a) \neq 0}$$

b) $x^2 - 3x + 2 = 0 \iff (x-2)(x-1) = 0$

$x_1 = 2, \ x_2 = 1$

c) $x^3 - x^2 - x + 1 = 0$

$x^2(x-1) - (x-1) = 0 \qquad (x-1)(x^2-1) = 0$

$(x-1)(x+1)(x-1) = 0 \qquad (x-1)^2(x+1) = 0$

$x_1 = -1 \qquad x_2 = 1$ dvojna

enojna (enostavna)

d) $x^3 - 8x = 0 \qquad x(x^2 - 8) = 0$

$x_1 = 0 \qquad x_{2,3} = \pm\sqrt{8} = \pm 2\sqrt{2}$

$x^2 = 8$

e) $x^3 - 6x^2 + 6x - 1 = 0$

$x^2(x-6) - (-6x+1) \dashrightarrow ??$

$x^3 - 1 - 6x(x-1) = 0$

$(x-1)(x^2+x+1) - 6x(x-1) = 0$

$$\boxed{\begin{aligned} a^3 - b^3 &= (a-b)(a^2+ab+b^2) \\ a^n - b^n &= (a-b)(a^{n-1}+a^{n-2}b+a^{n-3}b^2+\dots+b^{n-1}) \end{aligned}}$$

$$(x-1)(\underbrace{x^2+x+1-6x}_{x^2-5x+1})=0$$

$$\boxed{x_1=1} \quad \boxed{x_{2,3}}=\frac{-b\pm\sqrt{D}}{2a}=\boxed{\frac{5\pm\sqrt{21}}{2}}$$

$$D=b^2-4ac=25-4=21$$

13. Okrajšaj naslednja ulomka.

(a) $\frac{2x^2-5x+2}{x^2-8x+12}$

(b) $\frac{16x^2-1}{16x^2-8x+1}$

$1 = 2 \cdot \frac{1}{2} \qquad \frac{5}{2} = 2 + \frac{1}{2}$

a) $$\frac{2x^2-5x+2}{x^2-8x+12} = \frac{2\left(x^2-\frac{5}{2}x+1\right)}{(x-2)(x-6)} = \frac{2(x-2)\left(x-\frac{1}{2}\right)}{(x-2)(x-6)} =$$

$$= \frac{\cancel{(x-2)}(2x-1)}{\cancel{(x-2)}(x-6)} = \frac{2x-1}{x-6}$$

b) $$\frac{16x^2-1}{16x^2-8x+1} = \frac{\cancel{(4x-1)}(4x+1)}{(4x-1)^{\cancel{2}}} = \frac{4x+1}{4x-1}$$

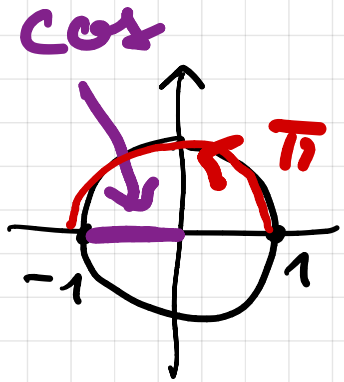
9. Reši enačbe.

(a) $\cos 3x + 1 = 0$

(b) $2\sin^2 x - \sin x = 0$

(c) $\sin x = \sin 2x$

a) $\cos 3x + 1 = 0 \iff \cos 3x = -1$

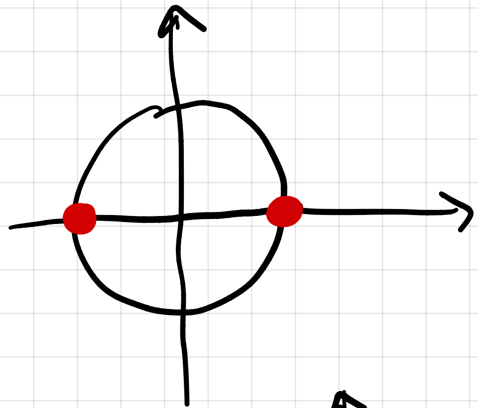


$3x = \pi + 2k\pi \,,\ k \in \mathbb{Z} \iff$

$\iff \boxed{x = \frac{\pi}{3} + \frac{2k\pi}{3} \,,\ k \in \mathbb{Z}}$

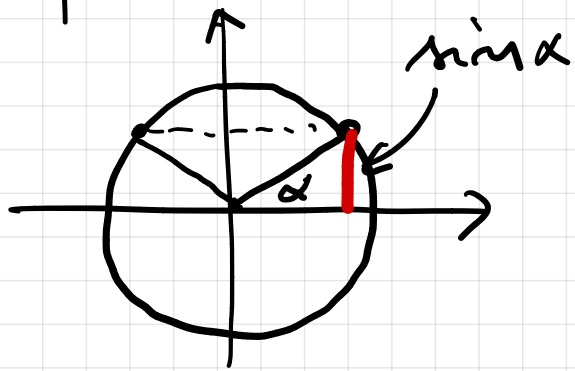
b) $2\sin^2 x - \sin x = 0$

$\sin x\,(2\sin x - 1) = 0$



① $\sin x = 0 \iff \boxed{x = k\pi \,,\ k \in \mathbb{Z}}$

② $2\sin x - 1 = 0 \iff \sin x = \frac{1}{2}$



$$\boxed{\begin{aligned} x_1 &= \frac{\pi}{6} + 2k\pi \,,\ k \in \mathbb{Z} \\ x_2 &= \frac{5\pi}{6} + 2k\pi \,,\ k \in \mathbb{Z} \end{aligned}}$$

α

$\pi - \frac{\pi}{6} = \frac{5\pi}{6}$

$\sin \alpha = \sin(\pi - \alpha)$

c) $\sin x = \sin 2x$

$\sin x = 2\sin x \cos x$

$\sin x - 2\sin x\cos x = 0$

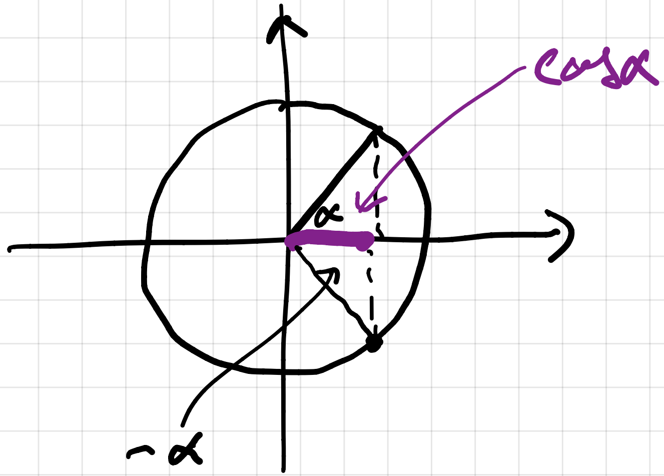
$\sin x\,(1-2\cos x) = 0$

① $\sin x = 0 \iff \boxed{x = k\pi,\ k\in\mathbb{Z}}$

② $1-2\cos x = 0 \iff \cos x = \frac{1}{2}$

$$\boxed{x_1 = \frac{\pi}{3} + 2k\pi,\ k\in\mathbb{Z}}$$

$$\boxed{x_2 = -\frac{\pi}{3} + 2k\pi,\ k\in\mathbb{Z}}$$



$\cos\alpha = \cos(-\alpha)$

10. Izrazi $\operatorname{tg}(x+y)$ s $\operatorname{tg} x$ in $\operatorname{tg} y$.

$$\operatorname{tg}(x+y) = \frac{\sin(x+y)}{\cos(x+y)} = \frac{\sin x \cos y + \cos x \sin y}{\cos x \cos y - \sin x \sin y} =$$

$$= \frac{\operatorname{tg} x \cos x \cos y + \cos x \cdot \operatorname{tg} y \cos y}{\cos x \cos y - \operatorname{tg} x \cos x \operatorname{tg} y \cos y} =$$

$$\left[\ \sin x = \operatorname{tg} x \cos x \qquad \sin y = \operatorname{tg} y \cos y \qquad \operatorname{tg} x = \frac{\sin x}{\cos x}\ \right]$$

$$= \frac{\cancel{\cos x \cos y}\,(\operatorname{tg} x + \operatorname{tg} y)}{\cancel{\cos x \cos y}\,(1 - \operatorname{tg} x \operatorname{tg} y)}$$

$$\boxed{\operatorname{tg}(x+y) = \frac{\operatorname{tg} x + \operatorname{tg} y}{1 - \operatorname{tg} x \operatorname{tg} y}}$$

14. Reši enačbe.

(a) $5^x + 5^{x+1} = 30$

(b) $9^x + 27 = 4 \cdot 3^{x+1}$

(c) $\log_{10}(2x+1) - \log_{10} 3 = 2\log_{10} 2 - \log_{10}(x+3)$

(d) $\log_x(\frac{1}{8}) = -3$

(e) $\log_x(2+\sqrt{3}) = -0.25$

(f) $\ln(2x-1) + \ln(x-2) = \ln x$

a) $5^x + 5^{x+1} = 30 = 5 \cdot 6$

$5^x(\underbrace{1+5}_{6}) = 5 \cdot 6 \Longrightarrow \boxed{x=1}$

"Po receptu": $t = 5^x$ - - →

b) $9^x + 27 = 4 \cdot 3^{x+1}$

$3^{2x} + 3^3 = 4 \cdot 3^{x+1}$

$3^{2x} - 4 \cdot 3^{x+1} = -3^3$, $t = 3^x$, $3^{x+1} = 3^x \cdot 3$

$t^2 - 4 \cdot t \cdot 3 = -27$

$t^2 - 12t + 27 = 0 \qquad (t-3)(t-9) = 0$

1) $t_1 = 3 = 3^x \Longrightarrow$ $\boxed{x_1 = 1}$

2) $t_2 = 9 = 3^x \Longrightarrow$ $\boxed{x_2 = 2}$

$\log_{10} 2^2$

(c) $\log_{10}(2x+1) - \log_{10} 3 = 2\log_{10} 2 - \log_{10}(x+3)$

$\boxed{\log_a b + \log_a c = \log_a bc}$

$\boxed{\log_a b - \log_a c = \log_a \frac{b}{c}}$

$\boxed{\alpha \log_a b = \log_a b^\alpha}$

$a, b, c > 0$

$$\log_{10} \frac{2x+1}{3} = \log_{10} \frac{4}{x+3} \quad \Big\}\ 10^{\cdots}$$

$$\frac{2x+1}{3} = \frac{4}{x+3}$$

$(2x+1)(x+3) = 12 \qquad 2x^2 + 7x - 9 = 0$

$D = 49 + 72 = 121$

$$x_{1,2} = \frac{-7 \pm 11}{4} \qquad x_1 = 1, \quad x_2 = -\frac{9}{2}$$

Preverimo ali sta argumenta $2x+1$ in $x+3$ pozitivna?

1) $x=1$ ✓: $2x+1,\ x+3>0$ ✓

2) $x=-\frac{9}{2}$: $2x+1=-8\ \underline{<0}$ //

d) $\log_x \frac{1}{8} = -3$

$$\boxed{\log_a b = c \iff a^c = b}$$

$$x^{-3} = \frac{1}{8} \iff x^{-3} = 8^{-1} \quad \Big/ \ ^{-1}$$

$$\Downarrow$$

$$\frac{1}{x^3} = \frac{1}{8} \iff x^3 = \underset{\substack{\parallel \\ 2^3}}{8} \implies \boxed{x=2}$$

e) $\log_x(2+\sqrt{3}) = -0{,}25$

$$x^{-0{,}25} = 2+\sqrt{3} \iff x^{-1/4} = 2+\sqrt{3} \quad \Big/ \ (\,\dots)^{-4}$$

$$\boxed{a^{m/n} = \sqrt[n]{a^m}}$$

$$x = (2+\sqrt{3})^{-4} = \frac{1}{(2+\sqrt{3})^4}$$

$$(2+\sqrt{3})^4 = \left((2+\sqrt{3})^2\right)^2 = (\underbrace{4+4\sqrt{3}+3}_{7+4\sqrt{3}})^2 =$$

$$= 49 + 56\sqrt{3} + 48 = 97 + 56\sqrt{3}$$

$$x = \frac{1}{97+56\sqrt{3}} = \frac{97-56\sqrt{3}}{97^2 - 3\cdot 56^2}$$

(f) $\ln(2x-1) + \ln(x-2) = \ln x$

$$\ln\left((2x-1)(x-2)\right) = \ln x \implies$$

$$\implies (2x-1)(x-2) = x$$

$$2x^2 - 6x + 2 = 0 \implies x^2 - 3x + 1 = 0$$

$$x_{1,2} = \frac{3 \pm \sqrt{5}}{2}$$

1) $\boxed{x = \frac{3+\sqrt{5}}{2}}$: $\quad 2x-1>0, \; x-2 = \frac{3+\sqrt{5}-4}{2} = \frac{\sqrt{5}-1}{2} > 0,$

$x > 0$

2) $x = \frac{3-\sqrt{5}}{2}$: $\quad x-2 = \frac{3-\sqrt{5}-4}{2} = \frac{-1-\sqrt{5}}{2} < 0$

12. Skiciraj grafe naslednjih funkcij.

(a) $f(x) = x^2 - 2x + 1$

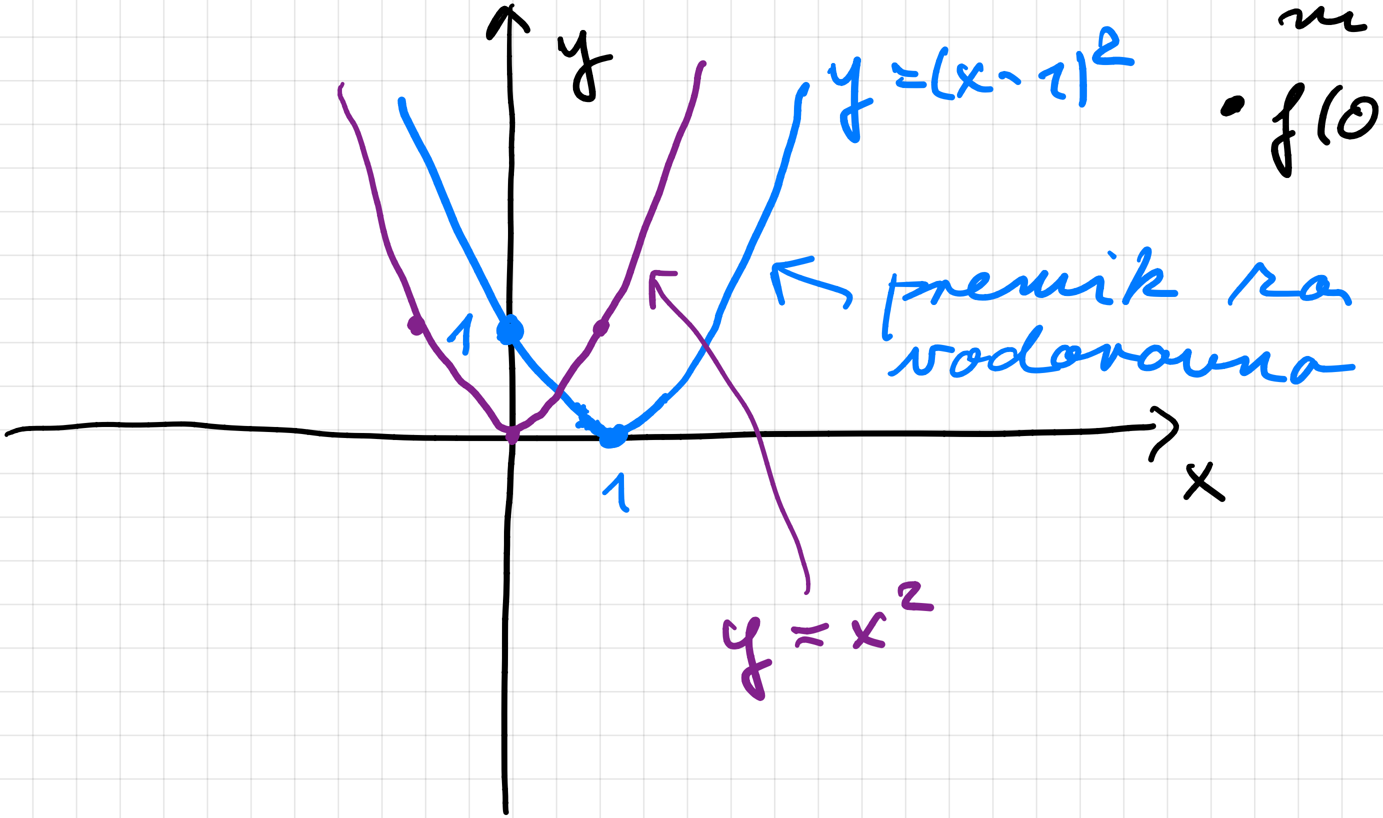
(b) $f(x) = x^2 - 3x + 2$

(c) $f(x) = x^3 - x^2 - x + 1$

(d) $f(x) = x^3 - 8x$

a) $y = f(x)$, $y = x^2 - 2x + 1 = (x-1)^2$

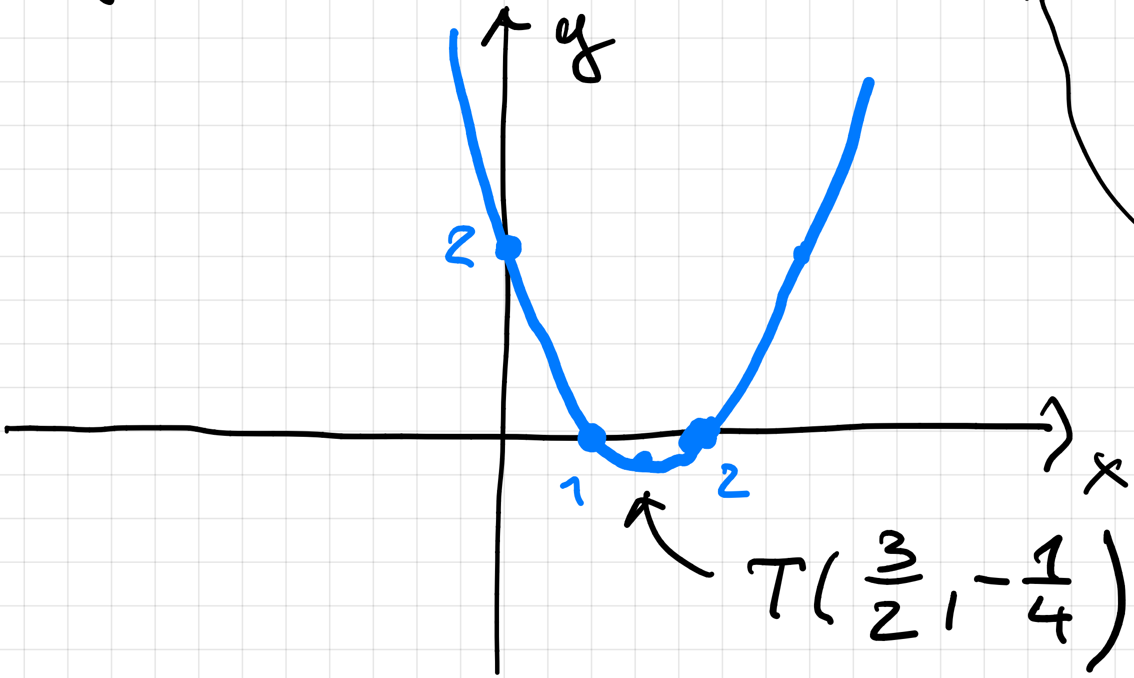
- $x = 1$ dvojna ničla
- $f(0) = 1$



b) $y = x^2 - 3x + 2 = (x-1)(x-2) = f(x)$

Teme: $x = \frac{3}{2}$

$f\left(\frac{3}{2}\right) = \frac{1}{2}\cdot\left(-\frac{1}{2}\right) = -\frac{1}{4}$



Kvadratna funkcija $f(x) = \overset{\neq 0}{a}x^2 + bx + c$

① Oblika za ničle: $f(x) = a\underbrace{(x-x_1)(x-x_2)}_{D \geq 0}$

② Temenska oblika: $f(x) = a(x-p)^2 + q$

Teme $T(p, q)$.

Naš primer

$$x^2 - 3x + 2 = \underbrace{x^2 - 3x + \left(\frac{3}{2}\right)^2}_{\text{KVADRAT}} - \left(\frac{3}{2}\right)^2 + 2 =$$

DOPOLNIMO DO KVADRATA

$$3x = 2 \cdot \frac{3}{2} \cdot x$$

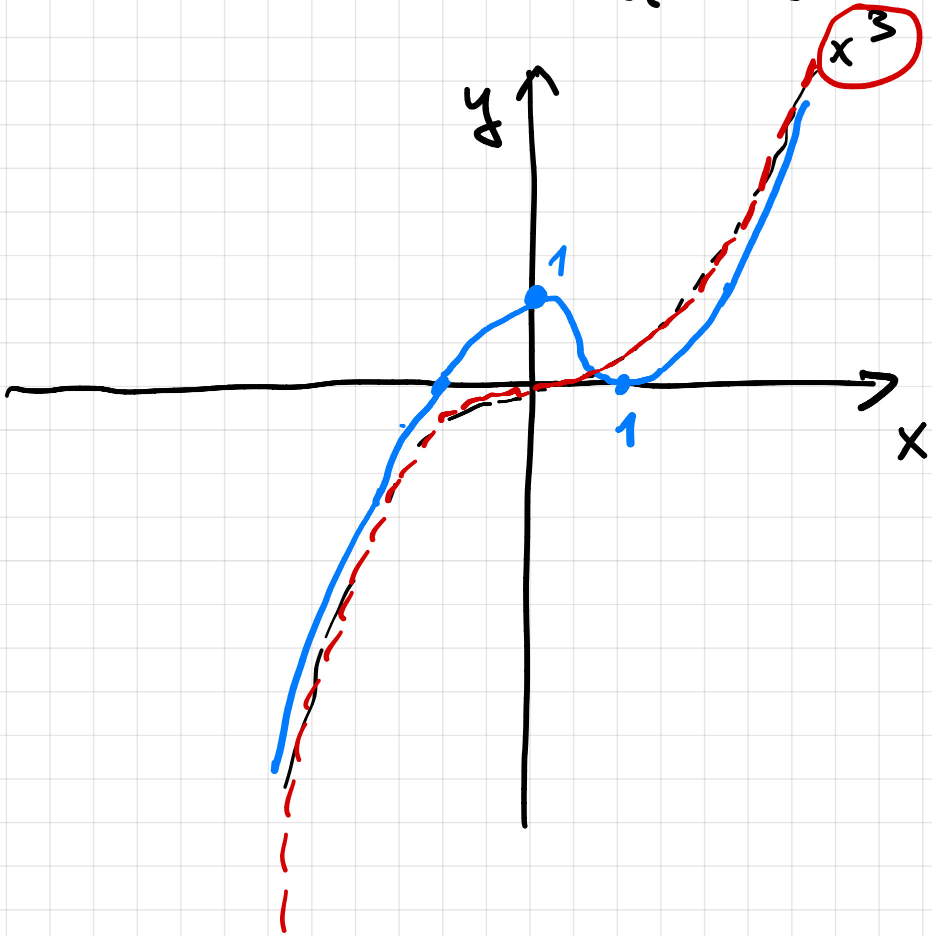
$$= \left(x - \frac{3}{2}\right)^2 - \frac{9}{4} + 2 = \left(x - \frac{3}{2}\right)^2 - \frac{1}{4}$$

$$T\left(\frac{3}{2}, -\frac{1}{4}\right)$$

c) $y = x^3 - x^2 - x + 1 = (x-1)^2(x+1)$

$x = 1$ dvojna ničla
$x = -1$ enostavna —||—] ničle

$f(0) = 1$

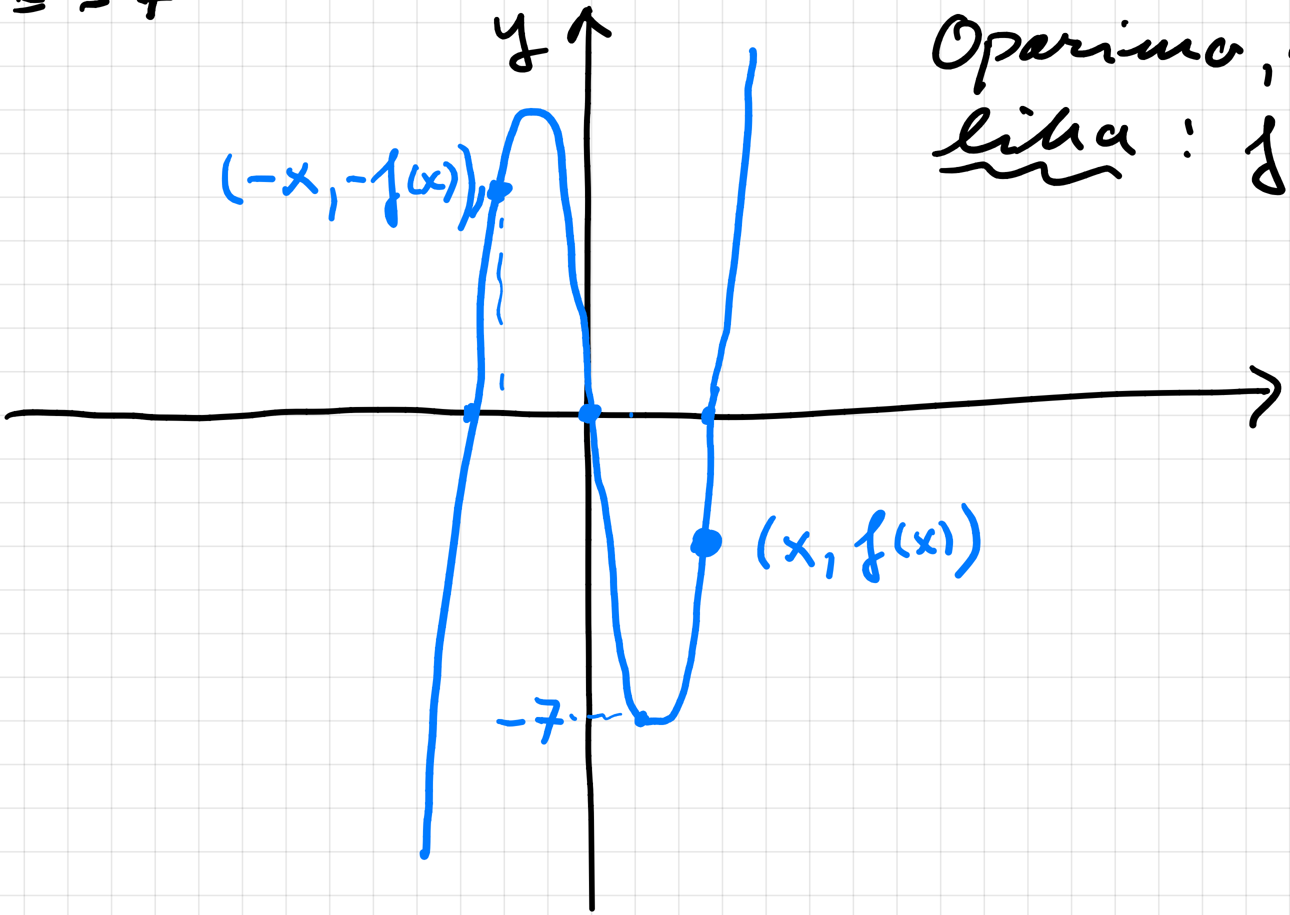


za velike x v izrazu prevlada člen x^3 : $f(x) \sim x^3$

d) $y = x^3 - 8x = x(x - 2\sqrt{2})(x + 2\sqrt{2})$

Ničle (enostavne): $x_1 = 0$, $x_2 = \underbrace{2\sqrt{2}}_{\doteq 2,8}$, $x_3 = -2\sqrt{2}$

$f(1) = -7$



Opazimo, da je $f(x)$ liha: $f(-x) = -f(x)$