Množice in izjavni račun

1. Naj bo $\mathcal{U} = \{1, 2, \ldots, 12\}$ univerzalna množica. Podani sta množici $A = \{1, 2, 4, 7, 9, 11\}$ in $B = \{2, 4, 5, 8, 9, 12\}$. Določi naslednje množice.

   (a) $A \cup B$,

   (b) $A \cap B$,

   (c) $A \backslash B$,

   (d) A^c.

   Naj bo podana še množica $C = \{3, 7, 9, 10\}$. Določi množico $(A \cap B^c) \cup C^c$.

a) $A \cup B = \{1, 2, 4, 7, 9, 11, 5, 8, 12\}$

b) $A \cap B = \{2, 4, 9\}$

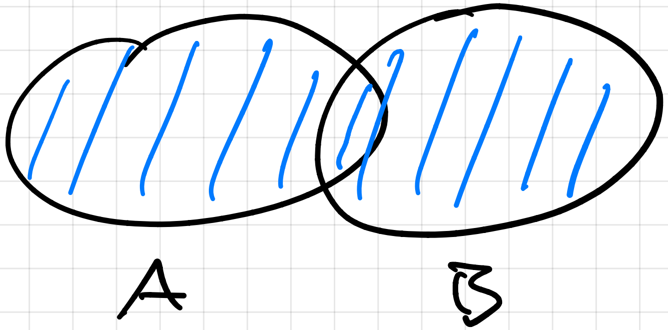
c) $A \setminus B = \{1, 7, 11\}$

"A brez B"

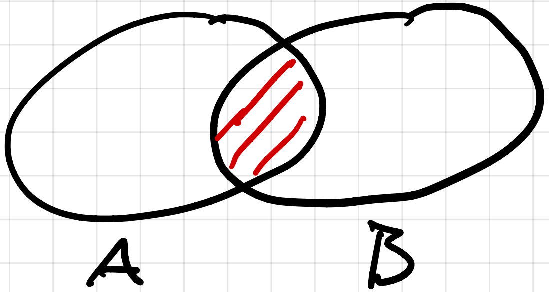
d) $A^c = U \setminus A = \{3, 5, 6, 8, 10, 12\}$

komplement A (poznati moramo U!!)

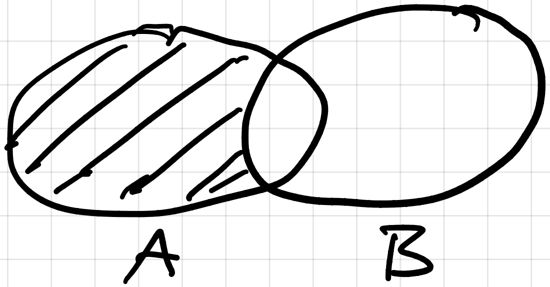
$A \cup B$:



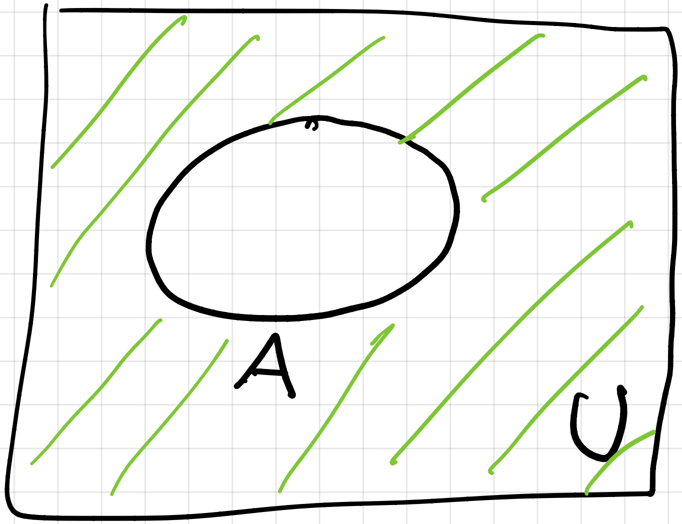
$A \cap B$:



$A \setminus B$:



A^c :



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Naj bo podana še množica $C = \{3, 7, 9, 10\}$. Določi množico $(A \cap B^c) \cup C^c$.

$$(A \cap B^c) \cup C^c = (A \cap \{1, 3, 6, 7, 10, 11\}) \cup$$

$$\cup \{1, 2, 4, 5, 6, 8, 11, 12\} =$$

$$= \{1, 7, 11\} \cup \{1, 2, 4, 5, 6, 8, 11, 12\} =$$

$$= \underline{\underline{\{1, 2, 4, 5, 6, 7, 8, 11, 12\}}}$$

2. Univerzalna množica naj bo množica naravnih števil. Podane so množice

$$A = \{n \in \mathbb{N} : \ n \text{ je deljivo s } 3\},$$

$$B = \{n \in \mathbb{N} : \ n \text{ je deljivo z } 2\},$$

$$C = \{n \in \mathbb{N} : \ n \text{ je deljivo s } 5\}.$$

Določi $A \cap B$, $A \cup B$, C^c, $(A \cap B) \cup C$ in $A \backslash C$. Za vsako od zgornjih množic napiši še prvih 5 elementov.

$A \cap B = \{n \in \mathbb{N} : n \in A \text{ in } n \in B\} =$

$= \{n : 3 \mid n \text{ in } 2 \mid n\} = \{n : 6 \mid n\}$

$6 = 3 \cdot 2$; ker sta 3 in 2 tuji števili (brez skupnega delitelja) je pogoj $(3 \mid n)$ in $(2 \mid n)$ enakovreden $6 \mid n$.

$A \cup B = \{n : n \in A \text{ ali } n \in B\} =$

$= \{n : 3 \mid n \text{ ali } 2 \mid n\} = \{2, 3, 4, 6, 8, 9, 10, 12, \ldots\}$

$C^c = \{n \in \mathbb{N} ; n \notin C\} = \{n : 5 \nmid n\} =$

$= \{1, 2, 3, 4, 6, 7, 8, 9, 11, 12, \ldots\}$

$$(A \cap B) \cup C = \{n : 6 \mid n\} \cup \{n : 5 \mid n\} =$$

$$= \{5, 6, 10, 12, 15, 18, \dots\}$$

$$A \setminus C = \{n : n \in A \text{ in } n \notin C\} \ (= A \cap C^c) =$$

$$= \{n : 3 \mid n \text{ in } 5 \nmid n\} = \{3, 6, 9, 12, 18, \dots\}$$

4. Naj bo $\mathcal{U} = \{1, 2, \ldots, 12\}$ univerzalna množica. Zapiši $(A \backslash C) \times D^c$ in $(A \cup C) \times (B \backslash D)$, kjer je $A = \{1, 2, 3, 4, 5\}$, $B = \{2, 4, 6, 8, 10, 12\}$, $C = \{2, 4\}$ in $D = \{6, 8, 12\}$.

$$\underline{(A \setminus C)} \times \underline{D^c} = \underline{\{1, 3, 5\}} \times \underline{\{1, 2, 3, 4, 5, 7, 9, 10, 11\}}$$

<u>kartezični produkt</u>

$$X \times Y = \{(x, y);\ x \in X,\ y \in Y\}$$

$$\begin{aligned}(A \setminus C) \times D^c = \{&(1,1), (1,2), (1,3), (1,4), (1,5), (1,7), \ldots\\ &(3,1), (3,2), \ldots\\ &(5,1), (5,2), \ldots, (5,11)\}\end{aligned}$$

$$(A \cup C) \times (B \setminus D) = \overbrace{\{1, 2, 3, 4, 5\} \times \{2, 4, 10\}}^{15 \text{ elementov}} =$$

$$= \{(1,2), (2,2), \ldots, (5,2), (1,4), \ldots, (5,4), (1,10), \ldots, (5,10)\}$$

3. Preveri, da veljata naslednji enakosti za množice A, B in C.

(a) $(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$,

(b) $(A \cup B)^c = A^c \cap B^c$.

a) $(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$

OPOMBA: $(a+b) \cdot c = ac + bc$ distributivnost

$(A \cup B) \cap C = \{x : x \in (A \cup B) \text{ in } x \in C\} =$

$= \{x : (x \in A \text{ ali } x \in B) \text{ in } x \in C\} =$

$= \{x ; (x \in A \text{ in } x \in C) \text{ ali } (x \in B \text{ in } x \in C)\} =$

$= \{x : x \in A \cap C \text{ ali } x \in B \cap C\} = \boxed{(A \cap C) \cup (B \cap C)}$ ✓

b) $(A \cup B)^c = \{x : x \notin A \cup B\}$

$(x \in A \cup B :$ $x \in A$ ali $x \in B)$ $\Longrightarrow$

$\Longrightarrow \underbrace{x \notin A \cup B : \text{ x ni niti v } A, \text{ niti v } B}_{x \notin A \text{ in } x \notin B}$

$\boxed{(A \cup B)^c} = \{x : x \notin A \text{ in } x \notin B\} =$

$= \{x : x \in A^c \text{ in } x \in B^c\} = \boxed{A^c \cap B^c}$