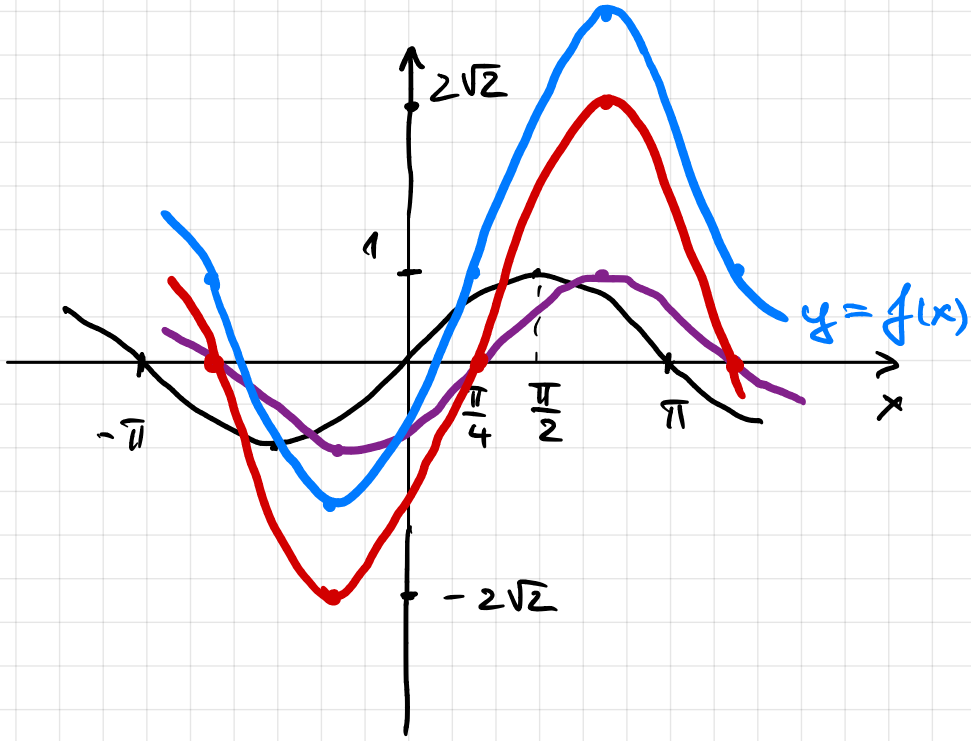
Izhodiščni graf: $y = \sin x$

$y = \sin(x - \frac{\pi}{4})$ ... premik za $\frac{\pi}{4}$ desno

$y = 2\sqrt{2}\sin(x - \frac{\pi}{4})$ ... razteg za $2\sqrt{2}$ navpično

$y = 2\sqrt{2}\sin(x - \frac{\pi}{4}) + 1 = f(x)$ ... premik za 1 gor



(c) $f(x) = x^3 - 3x^2 + 3x - 2$,

$D_f = \mathbb{R}$, ničle: $f(x) = (x-1)^3 - 1 = 0$

$\hookrightarrow x^3 - 3x^2 + 3x - 1$

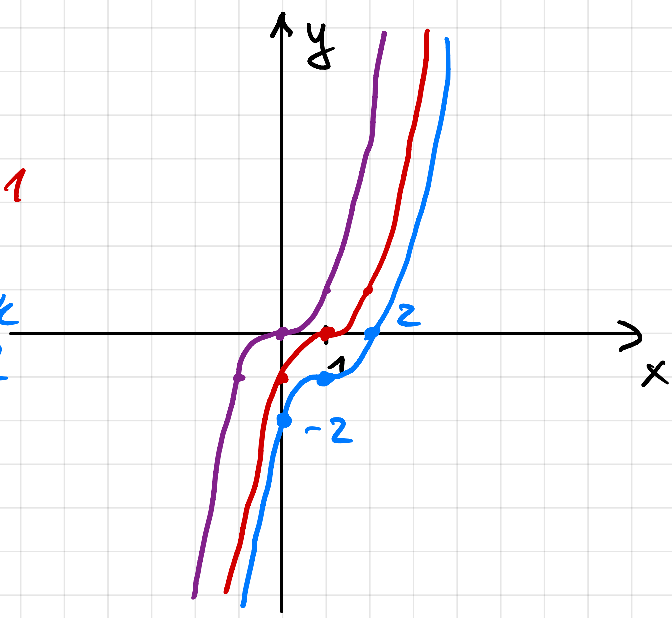
$(x-1)^3 = 1 \iff x - 1 = 1 \iff x = 2$

$\boxed{f(x) = (x-1)^3 - 1;}$

$g(x) = x^3$

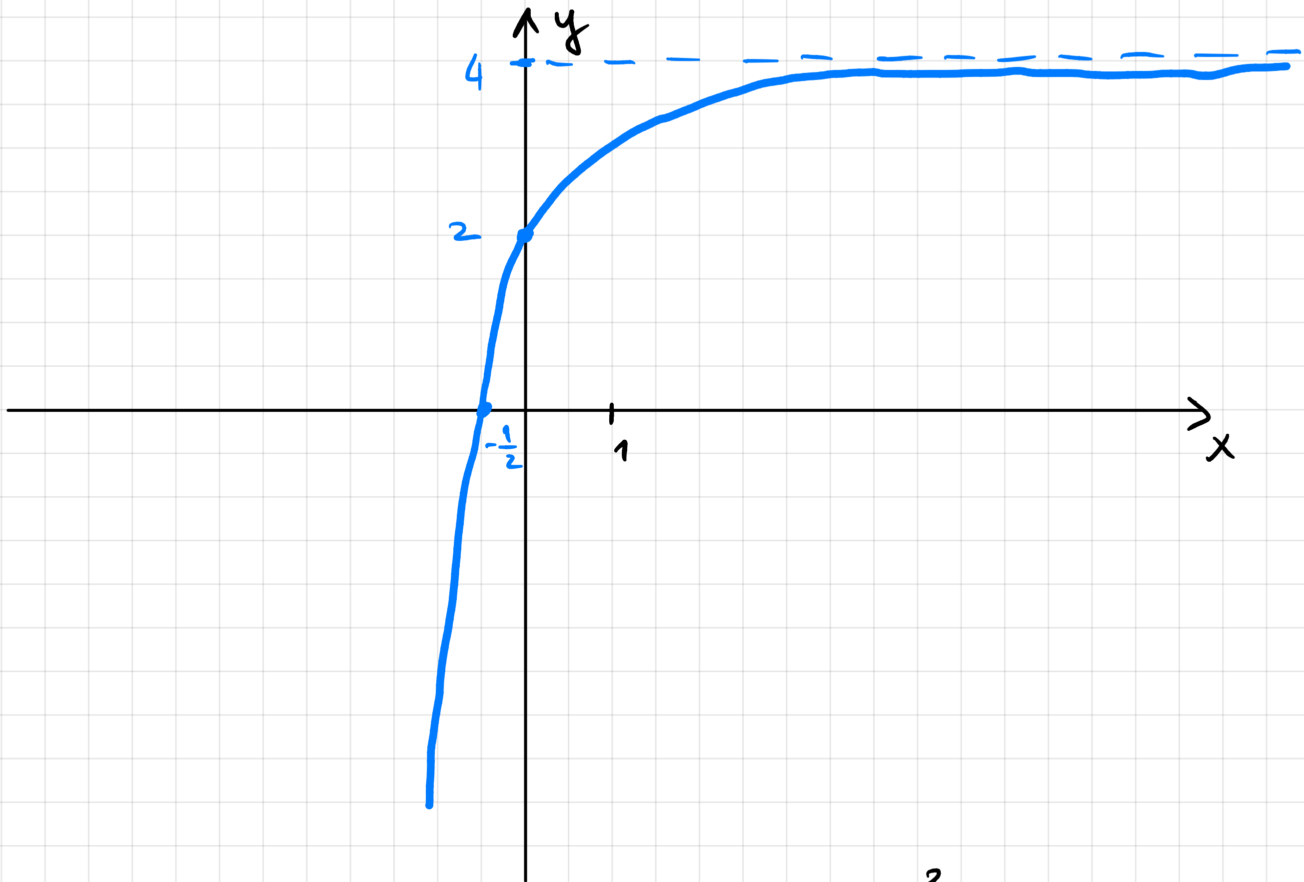
$h(x) = (x-1)^3$ ... premik 1 desno

$f(x) = h(x) - 1$ ... premik za 1 dol



(p) $f(x) = -4\left(\frac{1}{2^{2x+1}} - 1\right)$

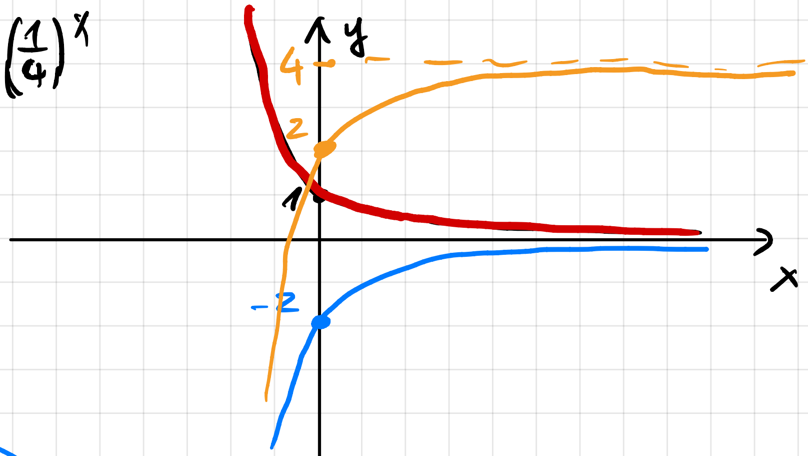
- $D_f = \mathbb{R}$
- Ničle: $\frac{1}{2^{2x+1}} - 1 = 0 \iff \frac{1}{2^{2x+1}} = 1 \iff$
  $\iff 2^{2x+1} = 1 \iff 2x+1=0 \iff x = -\frac{1}{2}$
- $$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} -4\Big(\underbrace{\frac{1}{\underbrace{2^{2x+1}}_{\to\infty}}}_{\to 0} - 1\Big) = 4$$
  $$\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty} -4\Big(\underbrace{\frac{1}{\underbrace{2^{2x+1}}_{\text{"}2^{-\infty}=0^+\text{"}}}}_{\to\infty} - 1\Big) = -\infty$$
- $f(0) = -4\left(\frac{1}{2} - 1\right) = -2 + 4 = 2$



2. način (z premiki / raztegi):

$$-4\left(\frac{1}{2^{2x+1}} - 1\right) = -4\cdot\frac{1}{2^{2x+1}} + 4 = -\overset{2^2}{4}\cdot 2^{-2x-1} + 4 =$$
$$= -2^{-2x+1} + 4 = -2\cdot 2^{-2x} + 4 = -2\cdot 4^{-x} + 4$$

$g(x) = 4^{-x} = \left(\frac{1}{4}\right)^x$



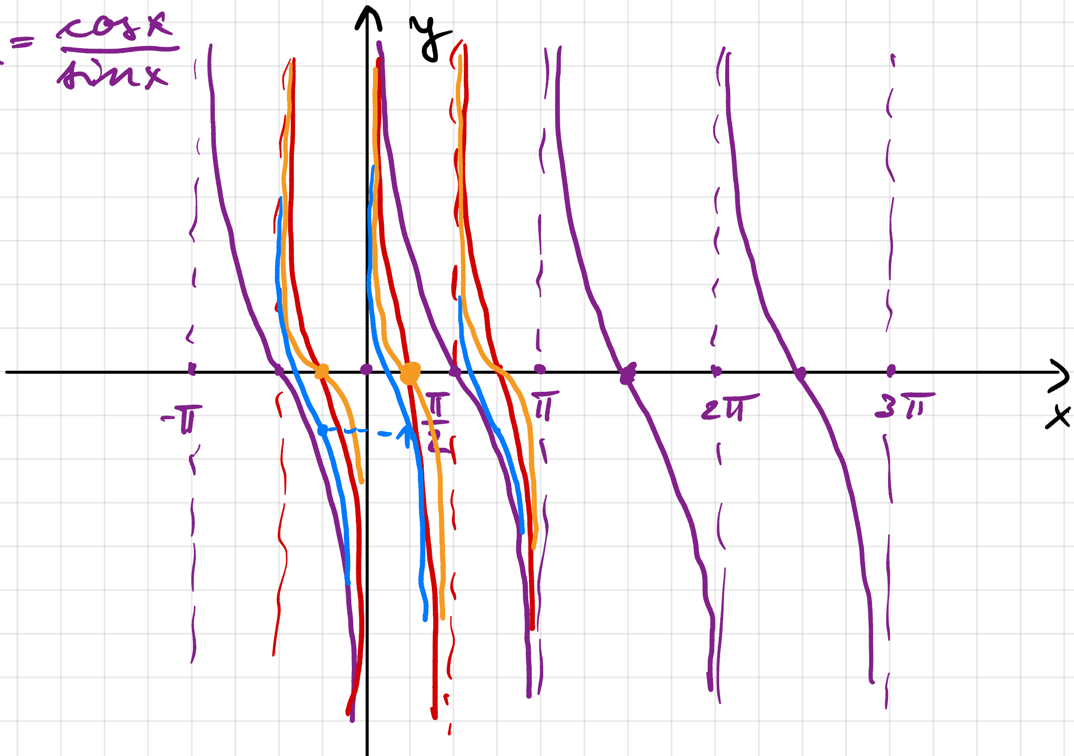
$h(x) = -2g(x)$ ... y koor. množimo z -2

(raztez za 2 in zrcaljenje čez x os)

$f(x) = h(x) + 4$ ... premik za 4 navzgor

(r) $f(x) = \frac{1}{2}\mathrm{ctg}(2x) - 1$

$g(x) = \mathrm{ctg}\, x = \frac{\cos x}{\sin x}$



$h(x) = \mathrm{ctg}\, 2x = \mathrm{ctg}\frac{x}{1/2}$ .... raztez za $1/2$ vodoravno

$g\left(\frac{x}{a}\right)$

$k(x) = \frac{1}{2}\mathrm{ctg}\, 2x = \frac{1}{2}h(x)$ ... raztez za $1/2$ navpično

$f(x) = k(x) - 1$ ... premik za 1 "dol".

Opomba $\mathrm{ctg}\, x = \frac{\cos x}{\sin x} = \frac{\sin(\frac{\pi}{2} - x)}{\cos(\frac{\pi}{2} - x)} = \mathrm{tg}(\frac{\pi}{2} - x) =$

$= -\mathrm{tg}(x - \frac{\pi}{2})$ (premik + zrcaljenje)

Funkcija f je podana s predpisom

$$f(x) = \ln \frac{1+2x}{1-x}.$$

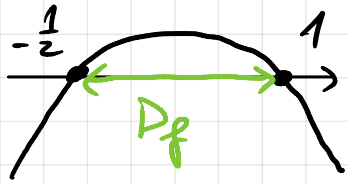
a) Določi definicijsko območje $\mathcal{D}_f$ in zalogo vrednosti $\mathcal{Z}_f$ funkcije f.

b) Obravnavaj injektivnost, surjektivnost in bijektivnost funkcije $f : \mathcal{D}_f \to \mathbb{R}$.

c) Izračunaj inverzno funkcijo funkciji $f : \mathcal{D}_f \to \mathcal{Z}_f$.

d) Skiciraj graf funkcije f.

a) D_f: $\frac{1+2x}{1-x} > 0 \quad \{ \cdot (1-x)^2 \quad \iff \quad \underbrace{(1+2x)(1-x)}_{x_1=-\frac{1}{2},\, x_2=1} > 0$



$D_f = (-\frac{1}{2}, 1)$

• Če se x bliža $-\frac{1}{2}$ se $\frac{\overset{\to 0}{1+2x}}{\underset{\to \frac{3}{2}}{1-x}}$ bliža 0

$\implies \ln \frac{1+2x}{1-x}$ se bliža $-\infty$

- Če se x bliža 1, se $\dfrac{1+2x}{1-x}$ bliža ∞ ($1+2x \to 3$, $1-x \to 0^+$)

$$\Longrightarrow \ln\frac{1+2x}{1-x} \text{ se bliža } \infty.$$

$$\Longrightarrow Z_f = (-\infty, \infty) = \mathbb{R}$$

b) Injektivnost: $f(a) = f(b) \stackrel{?}{\Longrightarrow} a = b$

$$\ln\frac{1+2a}{1-a} = \ln\frac{1+2b}{1-b} \Longrightarrow$$

$$\frac{1+2a}{1-a} = \frac{1+2b}{1-b}$$

$$(1+2a)(1-b) = (1+2b)(1-a)$$

$$\cancel{1} - b + 2a - \cancel{2ab} = \cancel{1} - a + 2b - \cancel{2ab}$$

$$3a = 3b \Longrightarrow a = b \Longrightarrow$$

$$\Longrightarrow f \text{ je injektivna}$$

Surjektivnost $f: D_f \to Z_f = \mathbb{R}$ ✓

Torej: $f: D_f \to Z_f$ je bijektivna

c) Inverzna funkcija

$$x = f(y) \leadsto y = f^{-1}(x)$$

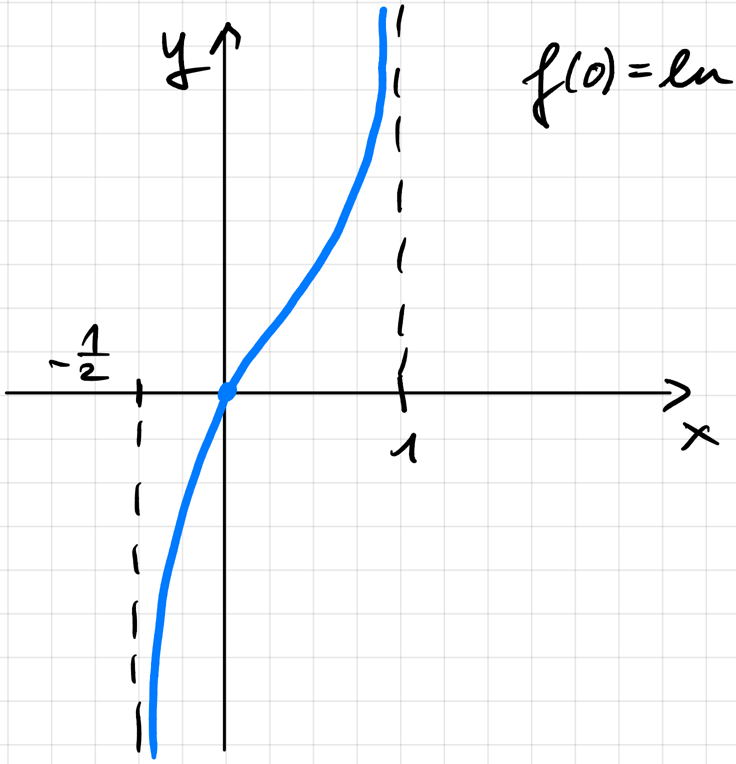
$$x = \ln\frac{1+2y}{1-y} \Longrightarrow e^x = \frac{1+2y}{1-y} \Longrightarrow$$

$$\Longrightarrow e^x - e^x y = 1 + 2y \Longrightarrow$$

$$e^x - 1 = y(e^x + 2) \Longrightarrow$$

$$y = \frac{e^x - 1}{e^x + 2} = f^{-1}(x)$$

d) Graf za f:



$$f(0) = \ln \frac{1+0}{1-0} = 0$$