(g) $$\lim_{x\to\infty} \underbrace{e^{-x}}_{\to 0}\ \underbrace{\sin x}_{\text{OMEJENO}} = 0$$

IZREK O SENDVIČU: $-e^{-x} \le e^{-x}\sin x \le e^{-x}$ $\to 0$

(h) $$\lim_{x\to 0}\left(\frac{1+\operatorname{tg} x}{1-\operatorname{tg} x}\right)^{1/x} \overset{1^{\pm\infty}}{=} \lim\left(\frac{1-\operatorname{tg} x+2\operatorname{tg} x}{1-\operatorname{tg} x}\right)^{1/x} =$$

$$= \lim_{x\to 0}\left(1+\frac{2\operatorname{tg} x}{1-\operatorname{tg} x}\right)^{1/x} = \lim_{x\to 0}\left(1+\frac{1}{(1-\operatorname{tg} x)/2\operatorname{tg} x}\right)^{1/x}$$

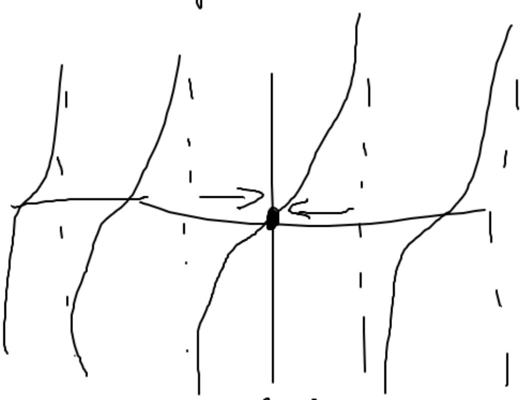
$$\boxed{\lim\left(1+\frac{1}{f(x)}\right)^{f(x)} = e \qquad f(x)\to\pm\infty}$$

$$= \lim_{x\to 0}\left(\underbrace{\left(1+\frac{1}{\frac{1-\operatorname{tg} x}{2\operatorname{tg} x}}\right)^{\frac{1-\operatorname{tg} x}{2\operatorname{tg} x}}}_{\to e}\right)^{\overbrace{\frac{2\operatorname{tg} x}{1-\operatorname{tg} x}\cdot\frac{1}{x}}^{\to 2}} = \underline{\underline{e^2}}$$

$$\lim_{x\downarrow 0}\frac{1-\operatorname{tg} x}{2\operatorname{tg} x} = \frac{1}{+0} = +\infty$$

$$\lim_{x\uparrow 0}\frac{1-\operatorname{tg} x}{2\operatorname{tg} x} = \frac{1}{-0} = -\infty$$

$$\lim_{x\to 0}\frac{2\operatorname{tg} x}{(1-\operatorname{tg} x)\cdot x} = \lim_{x\to 0}\underbrace{\frac{2}{\cos x(1-\operatorname{tg} x)}}_{\to \frac{2}{1\cdot 1}}\cdot\underbrace{\frac{\sin x}{x}}_{\to 1} = 2$$



(i) $$\lim_{x\to\infty}\frac{2x^2+2}{7x^2+3\sin(x)} \overset{:x^2}{\underset{:x^2}{=}} \lim_{x\to\infty}\frac{2+\frac{2}{x^2}}{7+\frac{3\sin x}{x^2}} = \underline{\underline{\frac{2}{7}}}$$ ($\frac{2}{x^2}\to 0$, $\frac{3\sin x}{x^2}\to 0$)

(j) $$\lim_{x\to\infty}\frac{\sqrt{x^2+4}+x}{2x} \overset{:x}{\underset{:x}{=}} \lim\frac{\sqrt{1+\frac{4}{x^2}}+1}{2} = \frac{\sqrt{1}+1}{2} = \underline{\underline{1}}$$

(k) $$\lim_{x\to\infty}\frac{\sqrt{x^2+4}-x}{2x} \overset{\infty-\infty}{\underset{\infty}{=}} \lim\frac{(\sqrt{\ }-x)(\sqrt{\ }+x)}{2x(\sqrt{\ }+x)} = \lim\frac{\cancel{x^2}+4-\cancel{x^2}}{2x(\sqrt{x^2+4}+x)}$$

$$= \frac{4}{\infty\cdot(\infty+\infty)} = \frac{4}{\infty} = \underline{0}$$

$$\lim_{x\to\infty}\frac{\sqrt{1+\frac{4}{x^2}}-1}{2} = \frac{1-1}{2} = \underline{\underline{0}}$$

(l) $\lim\limits_{x\to\infty} x^2\sqrt{1+x^2}-x^3 \overset{\infty-\infty}{=} \lim\limits_{x\to\infty} x^2\left(\sqrt{x^2+1}-x\right)$

$$= \lim_{x\to\infty} x^2 \frac{(\sqrt{\ }-x)(\sqrt{\ }+x)}{(\sqrt{\ }+x)} = \lim_{x\to\infty} x^2 \frac{\cancel{x^2}+1-\cancel{x^2}}{\sqrt{x^2+1}+x}$$

$$= \lim_{x\to\infty} \frac{x^2}{\sqrt{x^2+1}+x} \overset{:x}{\underset{:x}{=}} \lim \frac{x}{\sqrt{1+\frac{1}{x^2}}+1} = \frac{\infty}{\sqrt{1}+1} = \underline{\underline{\infty}}$$

če $\lim\limits_{x\to a} f(x)=\infty$ in $0<C\le g(x)$ na vse x,

potem $\lim\limits_{x\to a} \underbrace{f(x)}_{\infty}\,\underbrace{g(x)}_{\ge C > 0} \ge \lim f(x)\cdot C = \infty\cdot C=\infty$

(m) $\lim\limits_{x\to\infty}\left(\frac{x^2+2}{x^2-1}\right)^{x^2+x} \overset{1^\infty}{=} \lim\left(\frac{x^2-1+3}{x^2-1}\right)^{x^2+x} = \lim\limits_{x\to\infty}\left(1+\frac{3}{x^2-1}\right)^{x^2+x}$

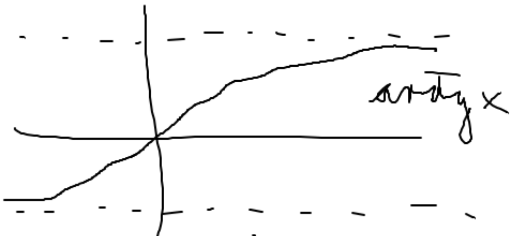
$$= \lim_{x\to\infty}\left(1+\frac{1}{(x^2-1)/3}\right)^{x^2+x} = \lim_{x\to\infty} \underbrace{\left(1+\frac{1}{(x^2-1)/3}\right)^{\frac{x^2-1}{3}}}_{e}{}^{\frac{3}{x^2-1}(x^2+x)}$$

$$= \lim_{x\to\infty} \underbrace{\left(1+\frac{1}{f(x)}\right)^{f(x)}}_{e}{}^{\overbrace{\frac{3(x^2+x)}{x^2-1}}^{\to 3}} = \underline{\underline{e^3}}$$

$f(x)\to\infty$

2. Ali je mogoče $f(x)$ dopolniti do funkcije, ki je zvezna povsod na $\mathbb{R}$?

(a) $f(x) = \operatorname{arctg}\left(\frac{1}{x}\right)$, $\quad \mathcal{D}_f = \mathbb{R}\setminus\{0\}$



f je zvezna povsod na $\mathbb{R}\setminus\{0\}$

Ali obstaja $C\in\mathbb{R}$, da bi bila tudi funkcija $\bar{f}(x) = \begin{cases} f(x); & x\neq 0 \\ C; & x = 0 \end{cases}$ ZVZ?

za ZVZ mora veljati $C = \lim\limits_{x\uparrow 0} f(x) = \lim\limits_{x\downarrow 0} f(x)$

$$\lim_{x\uparrow 0} f(x) = \lim_{x\uparrow 0} \underbrace{\operatorname{arctg}\big(\underbrace{\tfrac{1}{x}}_{\to -\infty}\big)}_{\operatorname{arctg}(-\infty)} = -\frac{\pi}{2}$$

$$\lim_{x\downarrow 0} f(x) = \lim_{x\downarrow 0} \underbrace{\operatorname{arctg}\big(\underbrace{\tfrac{1}{x}}_{\to +\infty}\big)}_{\operatorname{arctg}(+\infty)} = +\frac{\pi}{2}$$

$\neq \Rightarrow$ **NE**

(b) $f(x)=\frac{1-\cos x}{x^2}$, $D_f=\mathbb{R}\setminus\{0\}$

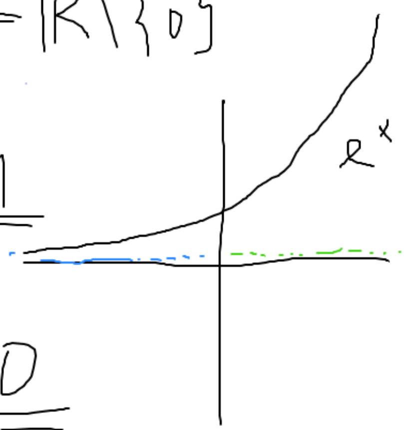
$$\lim_{x\to 0}\frac{1-\cos x}{x^2}\overset{\frac{+0}{+0}}{=}\lim\frac{(1-\cos x)(1+\cos x)}{x^2\,(1+\cos x)}=\lim_{x\to 0}\underbrace{\frac{\sin^2 x}{x^2}}_{\to 1}\cdot\frac{1}{\underbrace{(1+\cos x)}_{\to 2}}=\frac{1}{2}$$

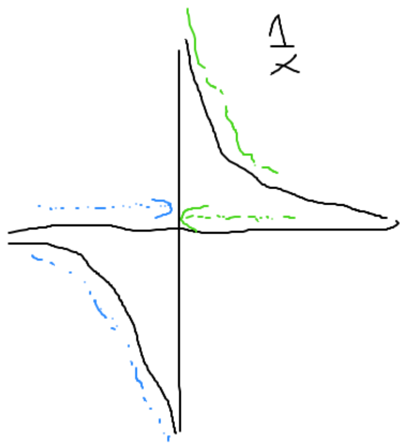
DA, $\bar{f}(x)=\begin{cases}\frac{1-\cos x}{x^2} & ;\ \text{če } x\neq 0\\ \frac{1}{2} & ;\ \text{če } x=0\end{cases}$ je ZVZ

L'HP

$$=\lim_{x\to 0}\frac{0-(-\sin x)}{2x}=\lim_{x\to 0}\frac{\sin x}{2x}\overset{L'HP}{\underset{\frac{0}{0}}{=}}\lim_{x\to 0}\frac{\cos x}{2}=\frac{1}{2}$$

(c) $f(x)=\frac{1}{1+e^{1/x}}$, $D_f=\mathbb{R}\setminus\{0\}$





$$\lim_{x\uparrow 0}\frac{1}{1+\underbrace{e^{\overbrace{1/x}^{\to -\infty}}}_{\to 0}}=\frac{1}{1+0}=1$$

$$\lim_{x\downarrow 0}\frac{1}{1+\underbrace{e^{\overbrace{1/x}^{\to +\infty}}}_{\to \infty}}=\frac{1}{1+\infty}=0$$

NE

(d) Pri katerih $a, b \in \mathbb{R}$ je $f(x) = \begin{cases} \frac{2}{\pi} \arcsin x ; & x \in [-1,1] \\ x^2 + ax + b ; & x \in \mathbb{R} \setminus [-1,1] \end{cases}$

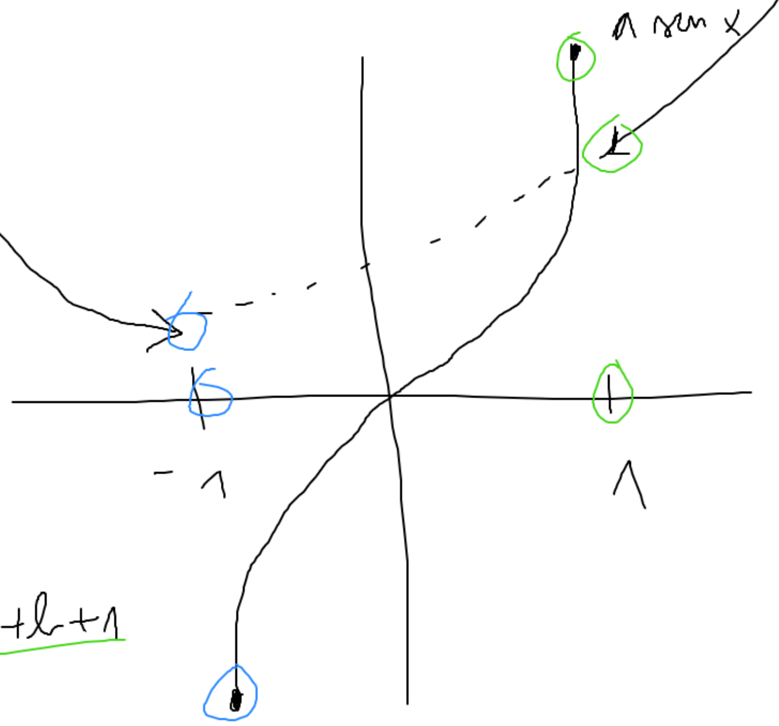
zvezna?

$$\lim_{x \uparrow -1} f(x) = (-1)^2 + a(-1) + b = -a + b + 1$$

$$\lim_{x \downarrow -1} f(x) = \frac{2}{\pi} \arcsin(-1) = \frac{2}{\pi}\left(-\frac{\pi}{2}\right) = -1$$

$$\lim_{x \uparrow 1} f(x) = \frac{2}{\pi}\left(\frac{\pi}{2}\right) = 1$$

$$\lim_{x \downarrow 1} f(x) = 1^2 + a \cdot 1 + b = a + b + 1$$



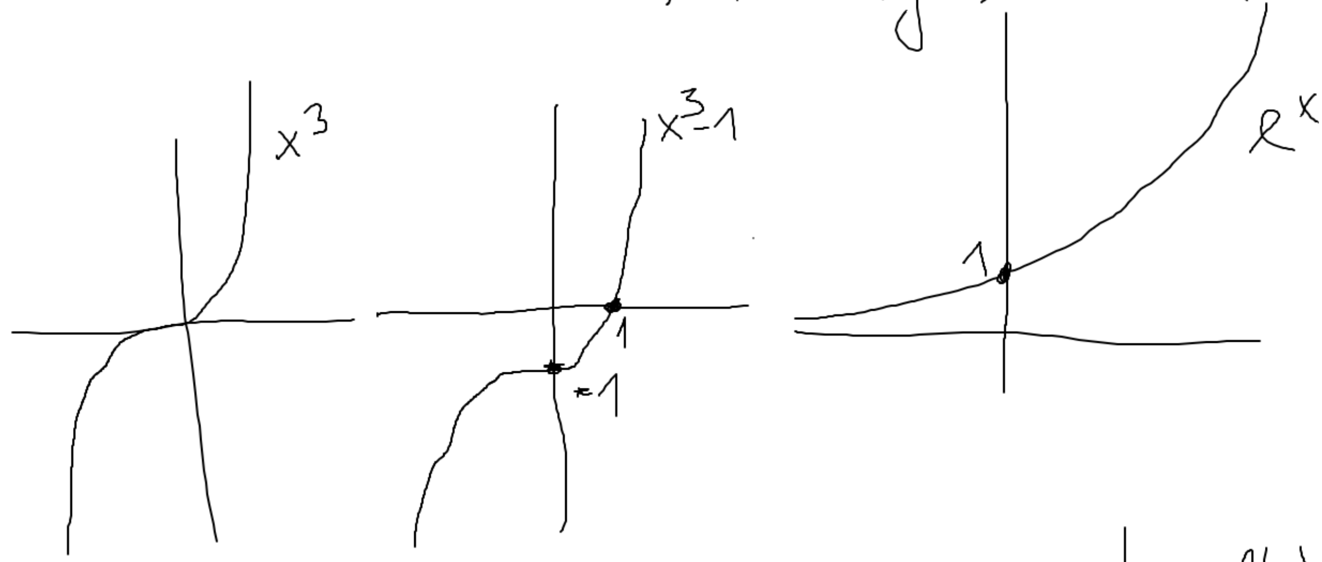
f zv z $\iff$ $-a + b + 1 = -1$

$a + b + 1 = 1$

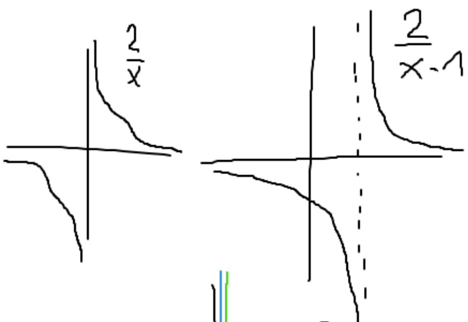
$(1) + (2): 2b + 2 = 0 \Rightarrow \underline{\underline{b = -1}}$

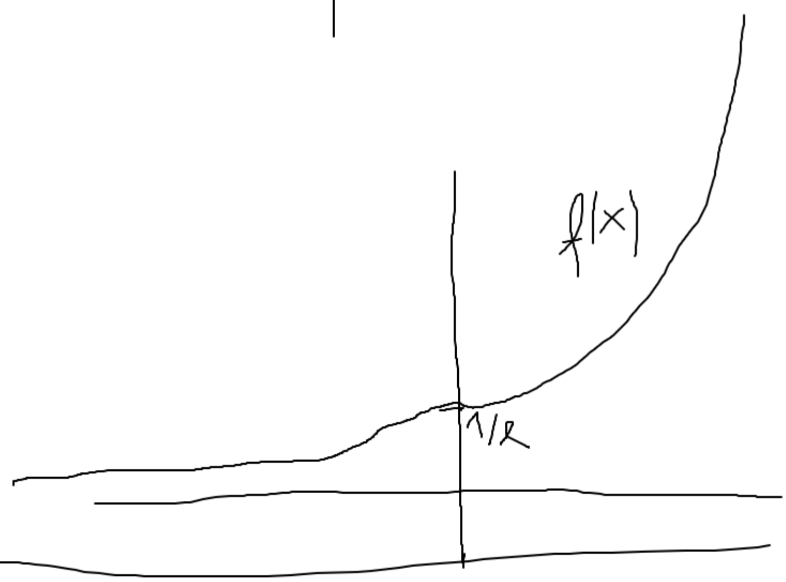
$(2) - (1): 2a = 2 \Rightarrow \underline{\underline{a = 1}}$

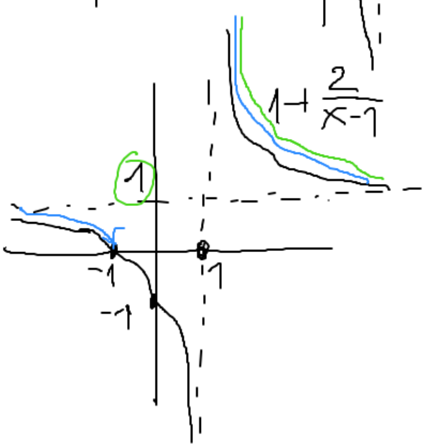
(e) Kot kompozitum nariši $f(x)=e^{x^3-1}$ in $g(x)=\sqrt{\ln\frac{x+1}{x-1}}$.

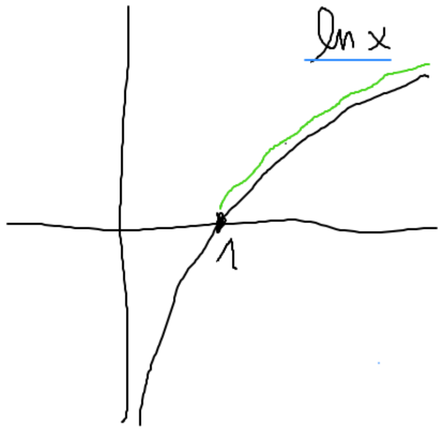


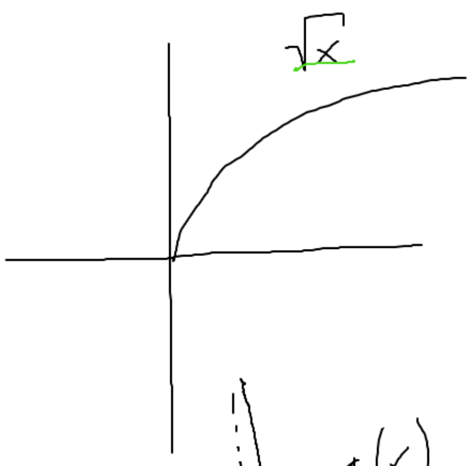
$$\frac{x+1}{x-1}=\frac{x-1+2}{x-1}=1+\frac{2}{x-1}$$











$\Rightarrow D_g=(1,\infty)$

na $(1,\infty)$ mora $1+\frac{2}{x-1}$ pada,

$\ln x$ narašča, $\sqrt{x}$ narašča

$\Rightarrow g$ pada

Ko $x\downarrow 1$, gre $1+\frac{2}{x-1}\to\infty \Rightarrow$

$\Rightarrow \ln(1+\frac{2}{x-1})\to\infty \Rightarrow \sqrt{\ln(\dots)}\to\infty$

Ko $x\to\infty$, gre

$1+\frac{2}{x+1}\to 1$

$\Rightarrow \ln(1+\frac{2}{x+1})\to 0$

$\Rightarrow \sqrt{\ln}\to 0$