(k) $\left(\frac{x\sin x}{2^x+\operatorname{tg}x}\right)' = \frac{(\sin x + x\cos x)(2^x+\operatorname{tg}x) - (x\sin x)(2^x\ln 2 + \frac{1}{\cos^2 x})}{(2^x+\operatorname{tg}x)^2}$

$a^{x\,\prime} = (e^{\ln a^x})' = (e^{x\ln a})' = \underline{e^{x\ln a}} \cdot \ln a = a^x \ln a$

(l) $(\cos x)^{\sin x\,\prime} = e^{\ln(\cos x^{\sin x})\,\prime} = (e^{\sin x\cdot\ln\cos x})' =$

$= \underbrace{e^{\sin x\cdot \ln\cos x}}_{(\cos x)^{\sin x}} \cdot \left(\cos x\cdot\ln\cos x + \sin x\cdot\frac{1}{\cos x}\cdot(-\sin x)\right)$

(m) $\arcsin(e^x)' = \frac{1}{\sqrt{1-e^{2x}}}\cdot e^x$

(n) $e^{\arcsin x\,\prime} = e^{\arcsin x}\cdot\frac{1}{\sqrt{1-x^2}}$

(o) $\sqrt{\frac{x}{e^x-1}}\,' = \frac{1}{2\sqrt{\frac{x}{e^x-1}}}\cdot\frac{(e^x-1)-xe^x}{(e^x-1)^2}$

(p) $\cos(e^{\sin\ln x})' = -\sin(e^{\sin\ln x})\cdot e^{\sin\ln x}\cdot\cos\ln x\cdot\frac{1}{x}$

(q) $\sqrt{\operatorname{tg}(x^2+\operatorname{arctg}x)}\,' = \frac{1}{2}(\operatorname{tg}(x^2+\operatorname{arctg}x))^{-1/2}\cdot\frac{1}{\cos^2(x^2+\operatorname{arctg}x)}\left(2x+\frac{1}{1+x^2}\right)$

(r) $\ln(\sin x)\cdot\sin(e^x)\cdot e^{\cos x\,\prime} =$

$= \left(\frac{1}{\sin x}\cdot\cos x\right)\sin e^x\cdot e^{\cos x} + \ln\sin x\cdot(\cos e^x\cdot e^x)\cdot e^{\cos x}$
$+ \ln\sin x\cdot\sin(e^x)\cdot(e^{\cos x}(-\sin x))$

$\boxed{2}$ (a) $f(x) = \underline{\sin(3x)}$, $f^{(51)}(x) = ? = 3^{51}\cdot(\underline{-\cos 3x})$

$f' = \cos(3x)\cdot 3^1$
$f'' = -\sin(3x)\cdot 3^2$
$f''' = \underline{-\cos(3x)}\cdot 3^3$
$f'''' = \underline{\sin(3x)}\cdot 3^4$

$\Big\}$ perioda odvajanja za $\sin x$ je 4

$51 = \underline{4\cdot 12} + \underline{3}$

(ã) $f(x) = \underline{\operatorname{ch}(5x)}$ $\qquad f^{(51)}(x) = \underline{\operatorname{sh}(5x)}\cdot 5^{51}$

$f' = \underline{\operatorname{sh}(5x)}\cdot 5$
$f'' = \underline{\operatorname{ch}(5x)}\cdot 5^2$

$\Big\}$ perioda $=2$

$51 = 2\cdot 25 + \underline{1}$

(b) $f(x) = e^{-x}\cdot x^2$, $f^{(10)}(x) = ?$

$$f' = e^{-x}\cdot(-1)x^2 + e^{-x}\cdot 2x$$
$$= e^{-x}(-x^2+2x)$$
$$f'' = e^{-x}\left((-1)(-x^2+2x)+(-x^2+2x)'\right) = e^{-x}(x^2-2x-2x+2)$$
$$= e^{-x}(x^2-4x+2)$$
$$f''' = e^{-x}(-x^2+4x-2+2x-4)$$
$$= e^{-x}(-x^2+6x-6)$$
$$f'''' = e^{-x}(x^2-6x+6-2x+6)$$
$$= e^{-x}(x^2-8x+12)$$

$f(x) = e^{-x}\cdot p(x)$, $p(x)$ = polinom

$$f' = e^{-x}(-1)\cdot p + e^{-x}\cdot p' = e^{-x}\overbrace{(-p+p')}^{=g}$$
$$f'' = e^{-x}(-g+g') = e^{-x}(p-p'-p'+p'') = e^{-x}\overbrace{(p-2p'+p'')}^{g}$$
$$f''' = e^{-x}(-g+g') = e^{-x}(-p+2p'-p''+p'-2p''+p''') = e^{-x}\overbrace{(-p+3p'-3p''+p''')}^{g}$$
$$f'''' = e^{-x}(p-3p'+3p''-p'''-p'+3p''-3p'''+p'''') = e^{-x}(p-4p'+6p''-4p'''+p'''')$$

$$f^{(m)}(x) \overset{?}{=} e^{-x}(-1)^m\sum_{k=0}^{m}\binom{m}{k}(-1)^k p^{(k)}$$

$$\binom{m}{k} = \begin{cases} \dfrac{m!}{k!\,(m-k)!}; & \text{če } m,k\in\mathbb{Z},\ 0\le k\le m \\ 0; & \text{sicer}\end{cases}$$

INDUKCIJA:

$m=0$: $f^{(0)}(x) = e^{-x}(-1)^0\cdot\binom{0}{0}(-1)^0 p^{(0)}$

$= e^{-x}p$ ✓

$$\binom{m+1}{k} = \binom{m}{k} + \binom{m}{k-1}$$

$m\Rightarrow m+1$: I.P. naj velja, RBD: $f^{(m+1)} \overset{?}{=} e^{-x}(-1)^{m+1}\sum_{k=0}^{m+1}\binom{m+1}{k}(-1)^k p^{(k)}$

$$f^{(m+1)} = \left(f^{(m)}\right)' \overset{\text{I.P.}}{=} \left(e^{-x}(-1)^m\sum_{k=0}^{m}\binom{m}{k}(-1)^k p^{(k)}\right)' =$$
$$= e^{-x}(-1)^{m+1}\sum_{k=0}^{m}\binom{m}{k}(-1)^k p^{(k)} + e^{-x}(-1)^m\cdot\sum_{k=0}^{m}\binom{m}{k}(-1)^k p^{(k+1)}$$
$$= e^{-x}(-1)^{m+1}\left(\sum_{k=0}^{m}\binom{m}{k}(-1)^k p^{(k)} - \sum_{k=1}^{m}\binom{m}{k-1}(-1)^{k-1}p^{(k)}\right)$$

$\binom{m}{m+1} = 0$

$\binom{m}{-1} = 0$

$$= e^{-x}(-1)^{m+1}\left(\sum_{k=0}^{m+1}\binom{m}{k}(-1)^k p^{(k)} + \sum_{k=0}^{m+1}(-1)^k\binom{m}{k-1}p^{(k)}\right)$$

$$= e^{-x}(-1)^{m+1}\sum_{k=0}^{m+1}\left(\binom{m}{k}+\binom{m}{k-1}\right)(-1)^k p^{(k)}$$

$$= e^{-x}(-1)^{m+1}\sum_{k=0}^{m+1}\binom{m+1}{k}(-1)^k p^{(k)}$$

□

$$\Rightarrow \left(e^{-x}p(x)\right)^{(m)} = e^{-x}(-1)^m\sum_{k=0}^{m}\binom{m}{k}(-1)^k p^{(k)}$$

$$= e^{-x}(-1)^m\left(p - \binom{m}{1}p' + \binom{m}{2}p'' - \binom{m}{3}p''' + \binom{m}{k}p''' - \dots \pm p^{(n)}\right)$$

ze $p = x^2$: $\left(e^{-x}x^2\right)^{(m)} = e^{-x}(-1)^m\left(p - mp' + \frac{m(m-1)}{2}p''\right)$

polinom stopnje d ima $p^{(d+k)} \equiv 0$

$$= e^{-x}(-1)^m\left(x^2 - m\cdot 2x + \frac{m(m-1)}{2}\cdot 2\right)$$

$$= e^{-x}(-1)^m\left(x^2 - 2mx + m(m-1)\right)$$

$m=1$: $e^{-x}(-x^2+2x)$

$m=2$: $e^{-x}(x^2-4x+2)$

$m=3$: $e^{-x}(-x^2+6x-6)$

$m=4$: $e^{-x}(x^2-8x+12)$

$\vdots$

$m=10$: $e^{-x}(x^2-20x+90)$

(c) $f(x) = x^{-1}, \quad f^{(n)} = ?$

$f' = -x^{-2}$

$f'' = -(-2)\, x^{-3}$

$f''' = (-1)(-2)(-3)\, x^{-4}$

$$\boxed{f^{(n)} = (-1)^n\, n!\, x^{-(n+1)}}$$

$\to$ INDUKCIJA:

$n=1$: $f^{(1)} = (-1)^1\, 1!\, x^{-2} = -x^{-2}$ ✓

$n \to n+1$: imamo: $f^{(n)} = (-1)^n\, n!\, x^{-(n+1)}$

RBD: $f^{(n+1)} \overset{?}{=} (-1)^{n+1} (n+1)!\, x^{-(n+2)}$

$$f^{(n+1)} = \left(f^{(n)}\right)' \overset{\text{I.P.}}{=} \left((-1)^n\, n!\, x^{-(n+1)}\right)'$$

$$= (-1)^n \cdot n!\, (-(n+1)) \cdot x^{-(n+1)-1}$$

$$= (-1)^{n+1} (n+1)!\, x^{-(n+2)}$$ ✓

(d) $f(x) = \frac{1}{x^2-1} = (x^2-1)^{-1}, \quad f^{(n)} = ?$

$f' = (-1)(x^2-1)^{-2} \cdot 2x$

$\quad = (-2x) \cdot (x^2-1)^{-2}$

$f'' = -2\,(x^2-1)^{-2} + (-2x)(-2)(x^2-1)^{-3} \cdot 2x$

$\quad = \left(-2\,(x^2-1) - 8x^2\right)(x^2-1)^{-3} = (-10x^2+2)\,(x^2-1)^{-3}$

$f = \frac{1}{p(x)} = p^{-1}$

$f' = (-1)p^{-2}p' = (-p')\,p^{-2}$

$f'' = (-p'')\,p^{-2} + (-p')(-2)p^{-3}p' =$

$\boxed{5.3.2021}$

$\boxed{2}$ (d) $f(x) = \frac{1}{x^2-1}$

$$= \frac{1}{(x-1)(x+1)} = \frac{A}{x-1} + \frac{B}{x+1}$$

$$= \frac{A(x+1)+B(x-1)}{(x-1)(x+1)} = \frac{x(A+B)+1(A-B)}{(x+1)(x-1)}$$

$1: \quad 1 = A-B \qquad (1)+(2): \quad 1 = 2A$

$x: \quad 0 = A+B \qquad A = \frac{1}{2},\ B = -A = -\frac{1}{2}$

$$\left(\frac{1}{x}\right)^{(n)} = \left(x^{-1}\right)^{(n)} = (-1)^n n!\, x^{-(n+1)}$$

$$f = \frac{1/2}{x-1} - \frac{1/2}{x+1} = \frac{1}{2}\left((x-1)^{-1} - (x+1)^{-1}\right)$$

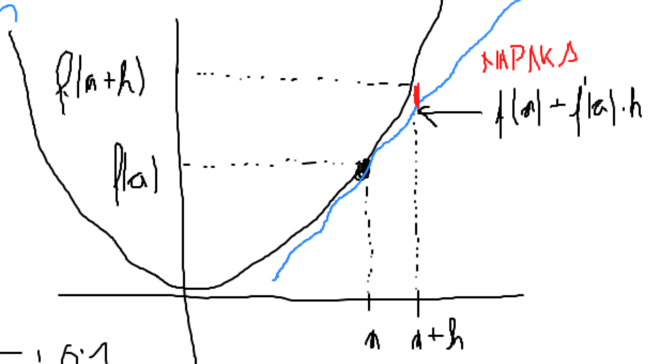
$$f^{(n)} = \frac{1}{2}\left((-1)^n n!\,(x-1)^{-(n+1)} - (-1)^n n!\,(x+1)^{-(n+1)}\right)$$

$$= \frac{(-1)^n n!}{2}\left((x-1)^{-(n+1)} - (x+1)^{-(n+1)}\right)$$

$\boxed{3}$ TAYLORJEVA FORMULA: $f(a+h) = \sum_{k=0}^{\infty} \frac{f^{(k)}(a)}{k!} h^k$

$$= f(a) + \frac{f'(a)}{1!}h^1 + \frac{f''(a)}{2!}h^2 + \frac{f'''(a)}{3!}h^3 + \dots$$

DIFERENCIAL: $f(a+h) \doteq f(a) + f'(a)\cdot h$



(a) $\ln(0.9) = ? = \ln(1-0.1) = f(0.1)$

$f(x) = \ln(1-x)$, $a = 0$, $h = 0.1$

$f'(x) = \frac{1}{1-x}\cdot(-1) = \frac{1}{x-1}$

$\Rightarrow f(0+0.1) \doteq f(0) + f'(0)\cdot 0.1 = \ln(1) + \frac{1}{0-1}\cdot 0.1 = \underline{\underline{-0.1}}$

(b) $\sqrt{\frac{0.9}{1.1}} = ? = \sqrt{\frac{1-0.1}{1+0.1}} = f(\underset{a}{0} + \underset{h}{0.1})$, $\quad f' = \frac{1}{2\sqrt{\ }}\cdot\frac{-(1+x)-(1-x)\cdot 1}{(1+x)^2} = \sqrt{\frac{1+x}{1-x}}\cdot\frac{-1}{(1+x)^2}$

$f(x) = \sqrt{\frac{1-x}{1+x}}$

$\Rightarrow f(0+0.1) \doteq f(0) + f'(0)\cdot 0.1 = 1 + (-1)\cdot 0.1 = \underline{\underline{0.9}}$

$(\hat{a})$ $\ln(0.9) = f(1+(-0.1)) \doteq f(1) + f'(1)\cdot(-0.1) = 0 + 1(-0.1) = \underline{\underline{-0.1}}$

$f(x) = \ln x$, $f' = \frac{1}{x}$

(c) $\sin\left(\frac{7\pi}{16}\right) = ? = \sin\left(\frac{(8-1)\pi}{16}\right) = \sin\left(\frac{\pi}{2} - \frac{\pi}{16}\right) = f\Big(\underset{a}{\underset{\parallel}{\frac{\pi}{2}}} + \underset{h}{\underset{\parallel}{\left(-\frac{\pi}{16}\right)}}\Big)$

$f(x) = \sin x, \quad f' = \cos x$

$\sin\left(\frac{7\pi}{16}\right) \doteq f\left(\frac{\pi}{2}\right) + f'\left(\frac{\pi}{2}\right) \cdot \left(-\frac{\pi}{16}\right) = \underline{\underline{1}} + 0 \cdot \left(-\frac{\pi}{16}\right)$

(d) $\operatorname{arctg}(\sqrt{3} + 0{,}01) = ? = f(\underset{a}{\underset{\parallel}{\sqrt{3}}} + \underset{h}{\underset{\parallel}{0{,}01}})$

$f(x) = \operatorname{arctg}(x)$

$f' = \frac{1}{1+x^2}$

$\doteq f(\sqrt{3}) + f'(\sqrt{3}) \cdot 0{,}01$

$= \underline{\underline{\frac{\pi}{3} + \frac{1}{1+3} \cdot 0{,}01}}$

(e) $\operatorname{arctg}\sqrt{2{,}99} = ?$

$f(x) = \operatorname{arctg}\sqrt{3-x}, \quad f' = \frac{1}{1+(3-x)} \cdot \frac{-1}{2\sqrt{3-x}}$

$= f(0 + 0{,}01) \doteq f(0) + f'(0) \cdot 0{,}01 = \underline{\underline{\operatorname{arctg}\sqrt{3} + \frac{1}{4} \cdot \frac{-1}{2\sqrt{3}} \cdot 0{,}01}}$

$f(x) = \operatorname{arctg}\sqrt{x}, \quad f' = \frac{1}{1+x} \cdot \frac{1}{2\sqrt{x}}$

$= f(3 - 0{,}01) \doteq f(3) + f'(3) \cdot (-0{,}01) = \underline{\underline{\operatorname{arctg}\sqrt{3} + \frac{1}{4} \cdot \frac{1}{2\sqrt{3}} \cdot (-0{,}01)}}$

4 $f(x) = (2x^2-3x+1)e^x$, tangenta/normala v $T(0,y_0)$

$= T(0,f(0)) = T(0,1)$

premica z naklonom k skozi (x_0,y_0):

$$\boxed{\frac{y-y_0}{x-x_0} = k} \rightsquigarrow y-y_0 = k(x-x_0)$$

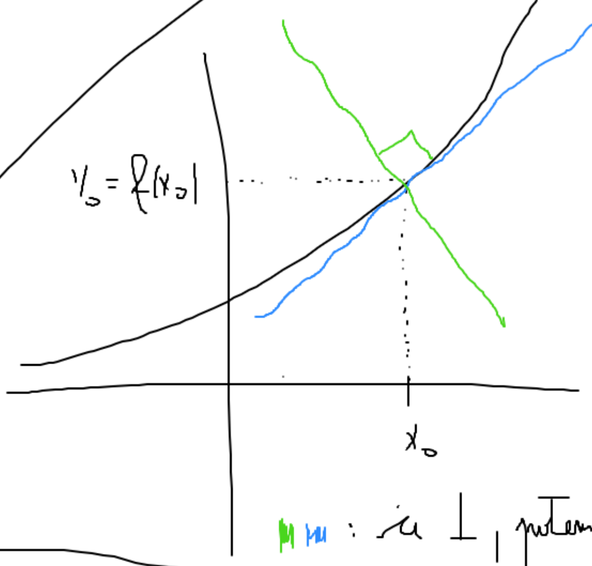
$$y = k(x-x_0)+y_0 = kx + \overbrace{(y_0-kx_0)}^{n}$$

$\Rightarrow$ tangenta na f v x_0:

$$\frac{y-f(x_0)}{x-x_0} = f'(x_0)$$

$\Rightarrow$ normala na f v x_0:

$$\frac{y-f(x_0)}{x-x_0} = \frac{-1}{f'(x_0)}$$



: je $\perp$, potem $k_2 = -\frac{1}{k_1}$

$$f' = e^x(2x^2-3x+1) + e^x(4x-3) = e^x(2x^2+x-2)$$

tangenta: $\frac{y-1}{x-0} = f'(0) = (-2) \Rightarrow y-1 = -2x \Rightarrow \underline{\underline{y = -2x+1}}$

normala: $\frac{y-1}{x-0} = \frac{-1}{f'(0)} = \frac{-1}{-2} \Rightarrow y-1 = \frac{x}{2} \Rightarrow \underline{\underline{y = \frac{x}{2}+1}}$

5 Kje je tangenta na $y = x^2-7x+3$ vzporedna na $y=-5x+3$.

Tangenta ima $k_1 = y' = 2x-7$

premica ima $k_2 = -5$

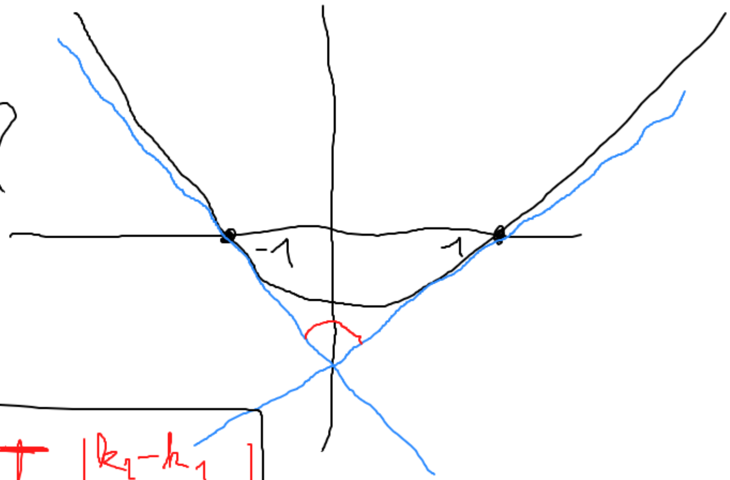
vzporedni sta, če $k_1 = k_2$

$$2x-7 = -5$$

$$\underline{\underline{x = 1}}$$

$\boxed{6}$ $f(x)=\frac{1}{2}(x^2-1)$

$\sphericalangle$(tangentama v $x=\pm 1$) $= ?$

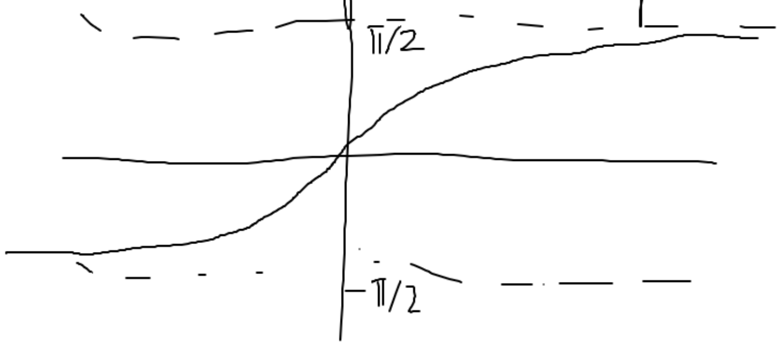


kot med premicama
$y=k_1x+m_1$ in
$y=k_2x+m_2$ je $\varphi=\operatorname{arctg}\left|\frac{k_2-k_1}{1+k_1\cdot k_2}\right|$

tangenta v $x=-1$: $k_1=f'(-1)=-1$ $\qquad f'=x$

tangenta v $x=1$: $k_2=f'(1)=1$

$$\varphi=\operatorname{arctg}\left|\frac{1-(-1)}{1-1}\right|=\operatorname{arctg}\left(\frac{2}{+0}\right)=\operatorname{arctg}(+\infty)=\underline{\underline{\frac{\pi}{2}}}$$



pravokotni-
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