9.3.2021

7 $y^4-4x^4-6xy=0$, tangenta v $T(1,2)$?

ali je T na krivulji? $2^4-4\cdot1^4-6\cdot1\cdot2=16-4-12=0$ ✓

$y=y(x)$ v neki okolici od T

za tangento rabimo $y'(1)=?$

$$y^4-4x^4-6x\cdot y=0 \;\Big/ \frac{d}{dx}$$

$$4y^3\cdot y'-16x^3-6y-6x\cdot y'=0$$

$$y'(4y^3-6x)=16x^3+6y$$

$$y'=\frac{16x^3+6y}{4y^3-6x} \qquad \Big/ \; x:=1,\; y:=2$$

$$y'(1)=\frac{16+6\cdot2}{4\cdot8-6}=\frac{28}{26}=\frac{14}{13}$$

$$4\cdot2^3y'-16-6\cdot2-6y'=0$$

$$y'(32-6)=16+12$$

$$y'=\frac{28}{26}=\frac{14}{13}$$

$\Downarrow$

tangenta:

$$\frac{y-y_0}{x-x_0}=y'(x_0)$$

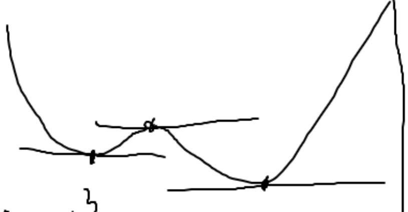
$$\frac{y-2}{x-1}=\frac{14}{13}$$

$$y=(x-1)\frac{14}{13}+2$$

KOLOKVIJ: pon. 12.4.2021,

8 $f(x)=x^2e^x$ na $[-4,3]$

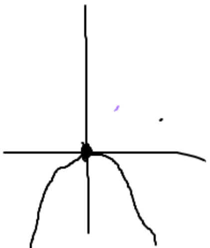
a stacionarna točka za f če $f'(a)=0$



$a=0$: $f=x^2$, $f'=2x$ | $f=-x^2$, $f'=-2x$ | $f=x^3$, $f'=3x^2$

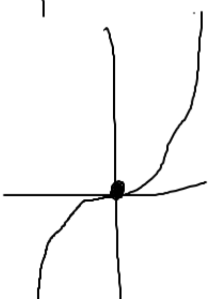
LOK. MIN

$f''=2$



LOK. MAX

$f''=-2$



SEDLO

$f''=6x$

$f''(0)=0$

v (0,0) ni tangente

$f(x)=x^2 e^x$

$f' = 2x e^x + x^2 e^x = \underline{\overbrace{(2x+x^2)}\, e^x}$ $\qquad f'=0 \Leftrightarrow x(x+2)=0$

$x=0,-2$

$f'' = (2+2x)e^x + (2x+x^2)e^x = (x^2+4x+2)e^x$ $\qquad$ dve stac. tč.

$f''(-2) = (4-8+2)e^{-2} < 0 \Rightarrow$  lok. max

$f''(0) = 2e^0 > 0 \Rightarrow$  lok. min

(in globalni)

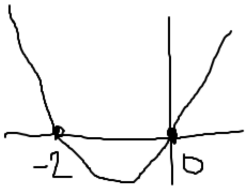
$D_f = \mathbb{R}$, ni soda, ni liha

$f(-x) \neq f(x) \qquad f(-x) \neq -f(x)$

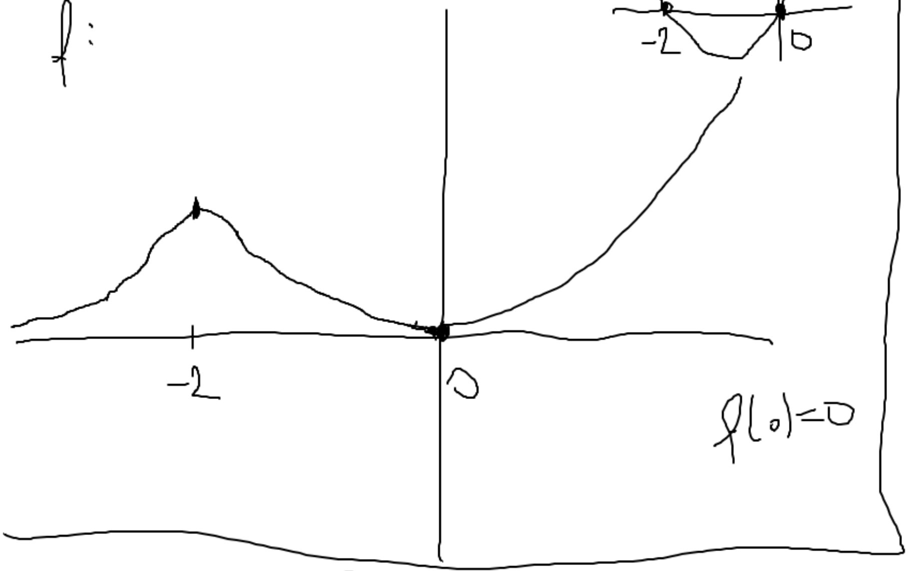
$$\lim_{x\to-\infty} x^2 e^x \underset{\substack{\infty\cdot 0\\ x=-t}}{=} \lim_{t\to\infty} t^2 e^{-t} = \lim_{t\to\infty} \frac{t^2}{e^t} \overset{2x\ L'HP}{=} \lim_{t\to\infty} \frac{2}{e^t} = \frac{2}{\infty} = 0$$

$$\lim_{x\to\infty} x^2 e^x = \infty\cdot\infty = \infty$$

naraščanje: $f'>0 \Leftrightarrow \underline{x(x+2)}\,\overset{\text{vedno poz.}}{\overline{e^x}} > 0 \Leftrightarrow x \in (-\infty,-2)\cup(0,\infty)$



f:



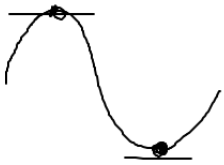
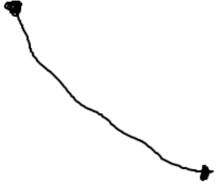
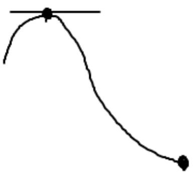
Kaj Z_f na $[-4,3]$?

za min in max preverimo stac. tč in krajišča

$f(-2)=\frac{4}{e^2}$, $f(0)=\underline{\underline{0}}$

$f(-4)=\frac{16}{e^4}$, $f(3)=\underline{\underline{9e^3}}$

$\Rightarrow Z_{f|_{[-4,3]}} = \underline{\underline{[0, 9e^3]}}$


$Z_{f|_{(-4,3)}} = [0, 9e^3)$

$\boxed{14}$ (a) $f = \frac{x}{x^2+1}$ $D_f = \mathbb{R}$

$$f' = \frac{1\cdot(x^2+1) - x\cdot(2x)}{(x^2+1)^2} = \frac{1-x^2}{(1+x^2)^2}$$

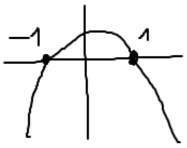
$$f'' = \frac{-2x(1+x^2)^2 - (1-x^2)\,2(1+x^2)\cdot 2x}{(1+x^2)^4} = \frac{2x(-1-2x^2\overbrace{-x^4-2+2x^4}{})}{(1+x^2)^4}$$

$$= \frac{2x(x^4-2x^2-3)}{(1+x^2)^4}$$

stac. tč.: $f' = 0 \iff 1-x^2=0 \iff (1-x)(1+x)=0 \iff x = \pm 1$

naraščanje: $f' > 0 \iff \frac{1-x^2}{\boxed{(1+x^2)^2}} > 0 \iff 1-x^2 > 0 \iff x \in (-1,1)$

vedno poz.



f narašča na $(-1,1)$

f pada na $(-\infty,-1)\cup(1,\infty)$

$\Longrightarrow$ v $x=-1$:  lok. min

v $x = 1$:  lok. max.

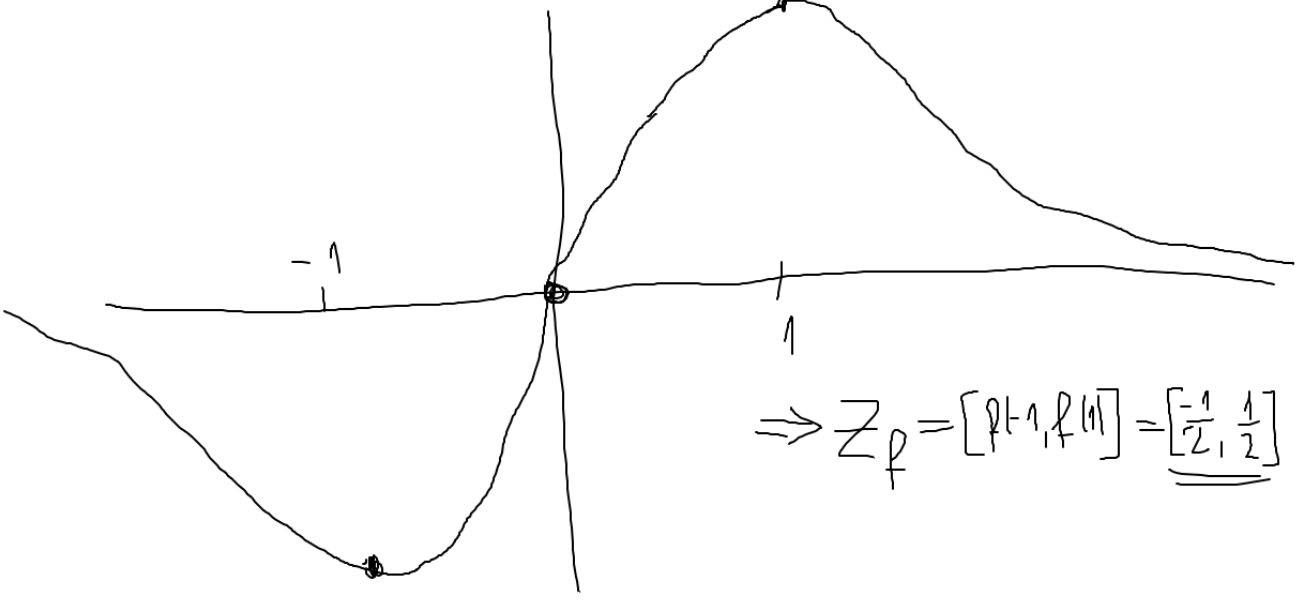
ALTERNATIVNO:

$f''(-1) = \frac{-2\cdot(-4)}{16} > 0 \Rightarrow$ 

$f''(1) = \frac{2(-4)}{16} < 0 \Rightarrow$ 

sodost/lihost: $f(-x) = \frac{-x}{(-x)^2+1} = -\frac{x}{x^2+1} = -f(x) \Rightarrow$ liha (graf je simetričen čez $(0,0)$)

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty}\frac{x}{x^2+1} = 0$$



$\Rightarrow Z_f = [f(-1), f(1)] = \underline{\underline{\left[\frac{-1}{2}, \frac{1}{2}\right]}}$

$\boxed{12.3.2021}$ K3 pet 16.4. ob 15h ⇐ do konca odvodov?

K4 tor 18.5. ob 16h

$\boxed{14}$ (b) $f(x) = e^x - x - 1$

- $D_f = \mathbb{R}$

POGLEDAMO ODVODE

- $f' = e^x - 1$, $f' = 0 \Leftrightarrow e^x = 1 \Leftrightarrow x = 0$

$f' > 0 \Leftrightarrow x > 0$

NARAŠČANJE/PADANJE

f na $(0, \infty)$ narašča, na $(-\infty, 0)$ pada

$\Rightarrow f(0) = e^0 - 0 - 1 = 0$ je minimum

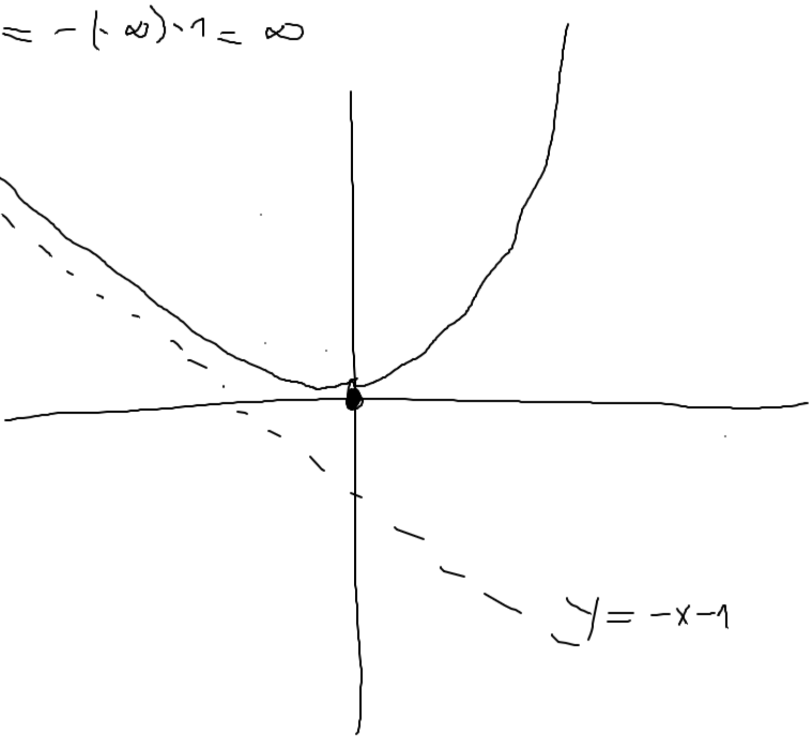
LIMITE:

$$\lim_{x \to \infty} \overset{\infty - \infty - 1}{e^x - x - 1} = \infty$$

$$\lim_{x \to \infty} \frac{e^x}{x} \overset{L'HP}{=} \lim_{x \to \infty} \frac{e^x}{1} = \frac{\infty}{1} = \infty$$

$$\lim \overset{\infty}{x}\Big(\overset{\infty}{\frac{e^x}{x}} - 1\Big) = \infty \cdot \infty = \infty$$

$$\lim_{x \to -\infty} \overset{0}{e^x} - \overset{-\infty}{x} - 1 = -(-\infty) - 1 = \infty$$

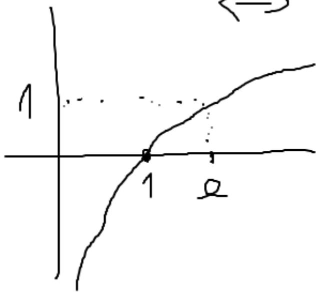


(c) $f(x)=\frac{x}{\ln x}$, D_f: $\ln x \neq 0 \Rightarrow x \neq 1 \Rightarrow D_f=(0,1)\cup(1,\infty)$

ODVOD: $f' = \frac{1\cdot \ln x - x\cdot \frac{1}{x}}{\ln^2 x} = \frac{\ln x - 1}{\ln^2 x}$

STACIONARNE TOČKE: $f'(x)=0 \Leftrightarrow \ln x - 1 = 0 \Leftrightarrow x = \underline{\underline{e}}$

NARAŠČANJE/PADANJE: $f' > 0 \Leftrightarrow \frac{\ln x - 1}{\underbrace{\ln^2 x}_{\text{vedno poz.}}} > 0 \Leftrightarrow \ln x - 1 > 0 \Leftrightarrow \ln x > 1$
$\Leftrightarrow x > e$



f na (e,∞) raste

f na $(0,1)$ in $(1,e)$ pada

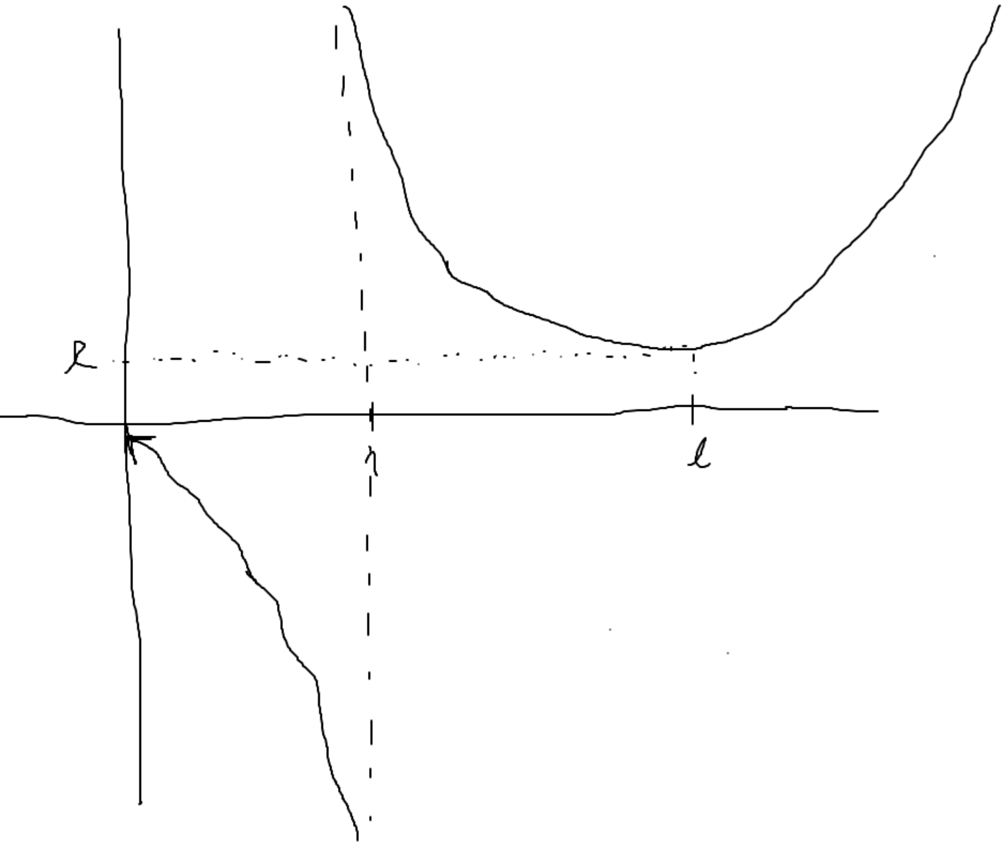
LIMITE: $\lim\limits_{x\downarrow 0} \frac{x}{\ln x} = \frac{0}{-\infty} = -0$

$\lim\limits_{x\uparrow 1} \frac{x}{\ln x} = \frac{1}{-0} = -\infty$

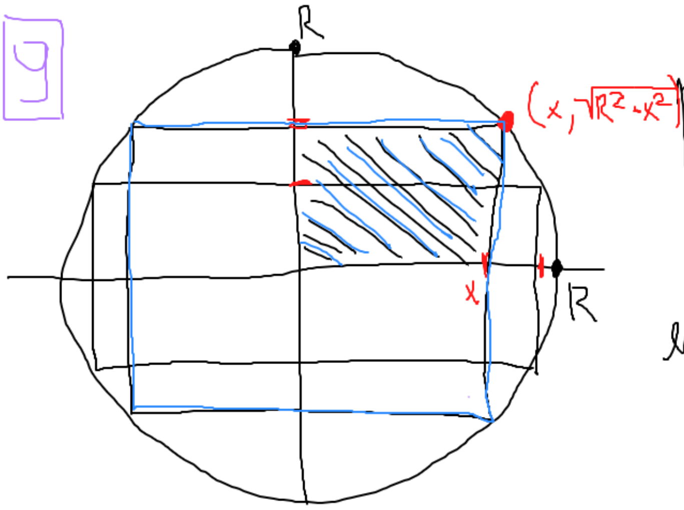
$\lim\limits_{x\downarrow 1} \frac{x}{\ln x} = \frac{1}{+0} = +\infty$

$\lim\limits_{x\to\infty} \frac{x}{\ln x} \overset{\infty/\infty}{\underset{L'HP}{=}} \lim\limits_{x\to\infty} \frac{1}{1/x} = \lim\limits_{x\to\infty} x = \infty$

$f(e) = \frac{e}{\ln e} = \underline{\underline{e}}$



$\boxed{9}$



pri katerem izbiri rdeče točke na krožnici je ploščina pravokotnika največja?

enačba krožnice: $x^2+y^2=R^2$

maksimiziramo $f(x) =$ ploščino pravokotnika

$$= 4x\cdot\sqrt{R^2-x^2}, \quad x \in [0,R]$$

ekstreme iščemo z odvodom:

$$f'(x) = 4\cdot 1\sqrt{R^2-x^2} + 4x\frac{1}{2\sqrt{\ }}\cdot(-2x)$$

$$= 4\left(\sqrt{\ } - \frac{x^2}{\sqrt{\ }}\right) = \frac{4}{\sqrt{R^2-x^2}}\left(\sqrt{\ }^{\,2} - x^2\right) = \frac{4}{\sqrt{R^2-x^2}}\left(R^2-2x^2\right)$$

STACIONARNE TOČKE: $R^2-2x^2=0 \Rightarrow x^2 = \frac{R^2}{2} \Rightarrow x = \frac{R}{\sqrt{2}}$

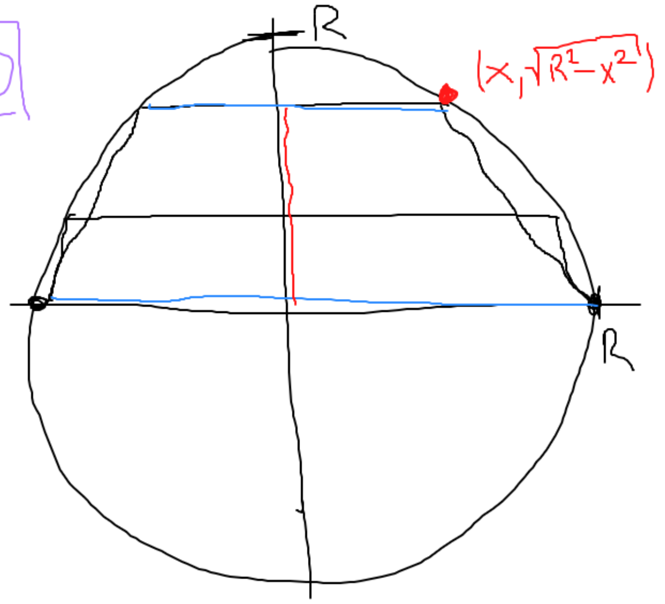
$$y = \sqrt{R^2-\frac{R^2}{2}} = \sqrt{\frac{R^2}{2}} = \underline{\underline{\frac{R}{\sqrt{2}}}}$$

VREDNOST f V KRAJIŠČIH: $f(0)=0$

$f(R)=0$

torej ploščina je največja, ko imamo <u>kvadrat</u>

$\boxed{10}$



pri katerem izboru • na krožnici je ploščina trapeza največja?

maksimiziramo $f(x)$ = ploščina trapeza

$$= \frac{\text{osnovnica 1} + \text{osnovnica 2}}{2} \cdot \text{višina}$$

$$= \frac{2R+2x}{2} \cdot \sqrt{R^2-x^2}$$

$$= (R+x)\cdot\sqrt{R^2-x^2}, \quad x \in [0,R]$$

ekstreme iščemo z odvodom: $f' = 1\cdot\sqrt{} + (R+x)\cdot\frac{1}{2\sqrt{}}(-2x)$

$$= \sqrt{} - (R+x)\frac{x}{\sqrt{}}$$

$$= \frac{1}{\sqrt{}}\left(\sqrt{}^{\,2} - (R+x)x\right)$$

$$= \frac{1}{\sqrt{R^2-x^2}}\left(R^2-x^2-Rx-x^2\right)$$

$$= \frac{1}{\sqrt{}}\left(R^2-Rx-2x^2\right)$$

→ stacionarne točke:

$$f' = 0 \Leftrightarrow -2x^2-Rx+R^2=0$$

$$x_{1,2} = \frac{R\pm\sqrt{R^2-4\cdot(-2)\cdot R^2}}{2(-2)}$$

$$= \frac{R\pm\sqrt{9R^2}}{-4} = \frac{3R\pm R}{4} \left\langle \begin{matrix} R \\ \frac{R}{2} \end{matrix} \right.$$

primerjam vrednost f v stac. tč. in krajiščih:

$$f(0) = (R+0)\sqrt{R^2-0} = \boxed{R^2}$$

$$f(R) = 0$$

$$f\left(\frac{R}{2}\right) = \left(R+\frac{R}{2}\right)\sqrt{R^2-\frac{R^2}{4}} = \frac{3}{2}R\cdot\sqrt{\frac{3R^2}{4}} = R^2\,\frac{3}{2}\cdot\frac{\sqrt{3}}{2} = R^2\,\frac{3\sqrt{3}}{4} > R^2$$

$$\Longrightarrow \boxed{x=\frac{R}{2}}$$