23.3.2021

PRIMERI, KJER PREDPOSTAVKE ZA L'HP NISO IZPOLNJENE:

• $\lim_{x\to\infty} \frac{x+\sin x}{x} = \lim_{x\to\infty}\left(1+\frac{\sin x}{x}\right) = 1+0$

$\frac{\sin x}{x} \to 0$, ker $\frac{-1}{x} \le \frac{\sin x}{\cos x} \le \frac{1}{x}$, $\to 0$

$\overset{\cancel{L'HP}}{=} \lim_{x\to\infty} \frac{1+\cos x}{1} \nexists$

ker $\lim_{x\to\infty}\frac{f'}{g'}$ ne obstaja, ne moremo sklepati $\lim\frac{f}{g}=\lim\frac{f'}{g'}$

• $\lim_{x\to\infty} \frac{\cancel{(x+\sin x\cos x)}}{\cancel{(x+\sin x\cos x)}\,e^{\sin x}}$ ← oscilira med e^{-1} in e^{1}, limita ne obstaja

$\overset{\cancel{L'HP}}{=} \lim_{x\to\infty}\frac{1+\cos^2 x-\sin^2 x}{(-\|-)e^{\sin x}+(x+\sin x\cos x)\cdot e^{\sin x}\cos x}$

$=\lim \frac{2\cos^2 x}{(2\cos^2 x+(x+\sin x\cos x)\cdot\cos x)\,e^{\sin x}} = \lim\frac{f'}{g'}$

$=\lim_{x\to\infty}\frac{\overbrace{2\cos x\cdot e^{-\sin x}}^{\text{omejeno na } [-2e,2e]}}{\underbrace{2\cos x+\sin x\cos x}_{\text{omejeno na } [-3,3]}+\underbrace{x}_{\to\infty}} = 0$

$g'\left(\frac{\pi}{2}+k\pi\right)=0$ za $k\in\mathbb{Z}$, torej $g'(x)\neq 0$ na nekem (A,∞) ne velja, zato ne velja $\lim\frac{f}{g}=\lim\frac{f'}{g'}$.

PRIMERI, KJER L'HP ZAKOMPLICIRA:

• $\lim_{x\to 0}\frac{x\sin x+\sqrt{1-2x^2}-1}{x^4} \overset{\frac{1-1}{0}}{=}$ $\qquad -2x(1-2x^2)^{-1/2}$

$=\lim_{x\to 0}\frac{\sin x+x\cos x+\overbrace{\frac{1}{2}(1-2x^2)^{-\frac{1}{2}}(-4x)}}{4x^3} \overset{\frac{0}{0}}{=}$ $\qquad -4x^2(1-2x^2)^{-3/2}$

$=\lim_{x\to 0}\frac{2\cos x - x\sin x - 2(1-2x^2)^{-1/2}+\overbrace{x(1-2x^2)^{-3/2}\cdot(-4x)}}{12x^2} \overset{\frac{2-2}{0}}{=}$

$=\lim_{x\to 0}\frac{-3\sin x - x\cos x + (1-2x^2)^{-3/2}\cdot(-4x) - 8x(1-2x^2)^{-3/2} - 4x^2\cdot\frac{-3}{2}(1-2x^2)^{-\frac{5}{2}}(-4x)}{24x}$

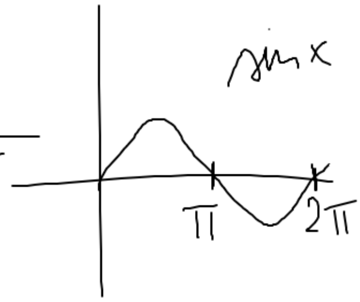
$=\frac{0}{0}$

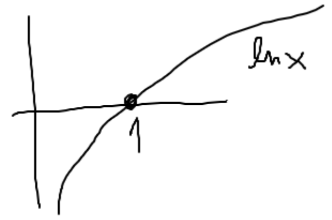
- $$\lim_{x\to 1} \frac{\ln x - \frac{2x-2}{x+1}}{\sin^3(\pi x)} \overset{\frac{0}{0}}{=} \lim \frac{\frac{1}{x} - \frac{2(x+1)-(2x-2)}{(x+1)^2}}{3\sin^2(\pi x)\cos(\pi x)\cdot\pi}$$

$$= \lim_{x\to 1} \frac{\frac{1}{x} - \frac{4}{(x+1)^2}}{3\pi \sin^2(\pi x)\cos(\pi x)} \overset{\frac{1-1}{0}}{=}$$

$$= \lim_{x\to 1} \frac{-x^{-2} - 4\cdot(-2)(x+1)^{-3}}{3\pi\, 2\sin(\pi x)\cos^2(\pi x)\,\pi - 3\pi\sin^3(\pi x)\cdot\pi}$$

$$= \frac{-1+8\cdot 2^{-3}}{0} = \frac{0}{0}$$





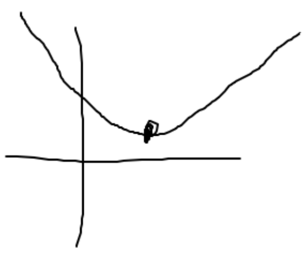
UPORABA ODVODA

① Določi števili a in b z vsoto 9, pri katerih je vrednost a^2+2b^2 minimalna.

$$a+b=9 \Rightarrow b=9-a \Rightarrow a^2+2b^2 = a^2+2\overbrace{(9-a)^2}^{81-18a+a^2} = a^2+2\cdot 9^2-36a+2a^2$$

$$=3a^2-36a+162 =: f(a)$$

minimiziramo $f(a)$ na $a\in\mathbb{R}$



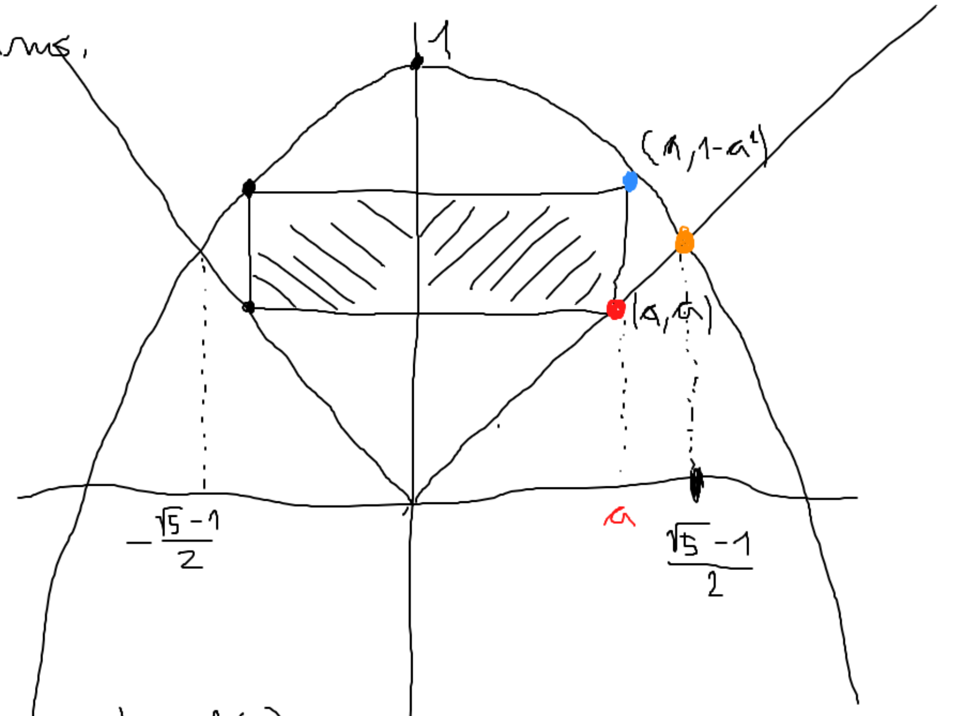
$$f(a)=3(a^2-12a+54)$$

$$f'(a)=3(2a-12)=0$$

$$\underline{\underline{a=6}} \Rightarrow \underline{\underline{b=3}}$$

Ker je f kvadratna funkcija ($\cup$ ali $\cap$), je dovolj poiskati le stacionarno točko (teme), sicer je treba izračunati tudi $\lim_{x\to-\infty} f(x)$ in $\lim_{x\to\infty} f(x)$

② V območju, omejeno s krivuljama $y = |x|$ in $y = 1 - x^2$ vriši pravokotnik (ki ima stranici vzporedni x- in y- osi) z največjo ploščino.

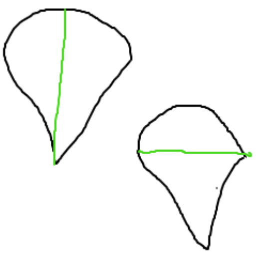


ploščina $= 2 \cdot a \cdot (1 - a^2 - a) =: f(a)$

maksimiziramo $f(a)$ na $a \in \left[0, \frac{\sqrt{5}-1}{2}\right]$

$f(0) = 0$

$f\left(\frac{\sqrt{5}-1}{2}\right) = 0$



$$1 - x^2 = x$$

$$x^2 + x - 1 = 0$$

$$x_{1,2} = \frac{-1 \pm \sqrt{1 - 4 \cdot (-1)}}{2} = \frac{-1 \pm \sqrt{5}}{2}$$

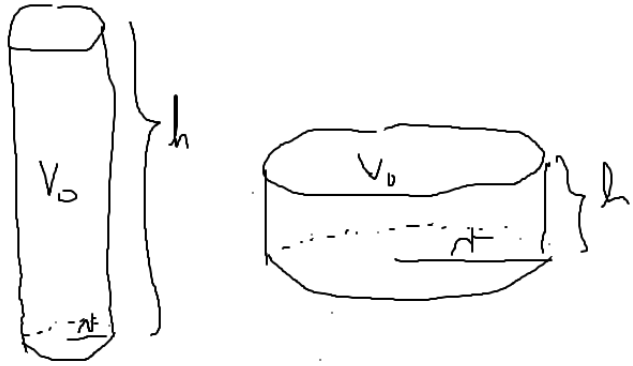
$$f(a) = 2(a - a^2 - a^3)$$

$$f' = 2(1 - 2a - 3a^2) = 0 \implies 3a^2 + 2a - 1 = 0$$

$$a_{1,2} = \frac{-2 \pm \sqrt{4 - 4 \cdot 3 \cdot (-1)}}{2 \cdot 3} = \frac{-2 \pm \sqrt{16}}{6} = \frac{-2 \pm 4}{6} = \begin{cases} -1 \notin D_f \\ \boxed{1/3} \end{cases}$$

26.3.2021

3. Posoda v obliki valja ima dan volumen V_0, je brez pokrova. Kakšno obliko naj ima, da bo poraba materiala minimalna?



minimiziramo površino posode pri fiksnem volumnu

$$V = \pi r^2 \cdot h = V_0 \Rightarrow h = \frac{V_0}{\pi r^2}$$

$$P = \pi r^2 + 2\pi r \cdot h = f(r, h)$$

$$\Rightarrow f(r) = \pi r^2 + 2\pi r \cdot \frac{V_0}{\pi r^2} = \pi r^2 + \frac{2V_0}{r}, \quad r \in (0, \infty)$$

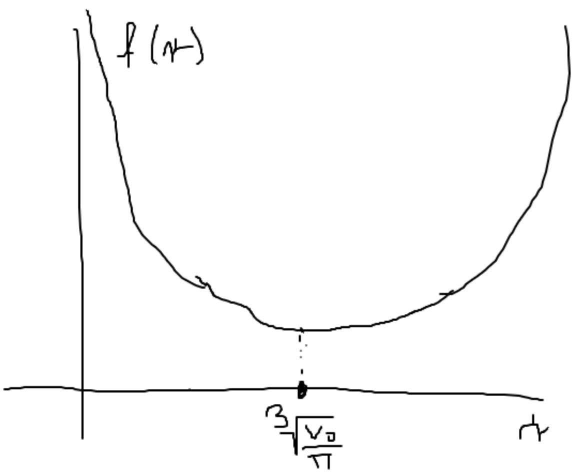
iščemo r, da je $f(r)$ minimalen

$$f'(r) = 2\pi r - 2V_0 r^{-2} = 2\,\frac{\pi r^3 - V_0}{r^2}$$

$$f' = 0 \Leftrightarrow \pi r^3 - V_0 = 0 \Leftrightarrow r = \sqrt[3]{\frac{V_0}{\pi}}$$

$$\lim_{r \downarrow 0} f(r) = \lim \pi r^2 + \frac{2V_0}{r} = 0 + \infty$$

$$\lim_{r \to \infty} f(r) = \lim \pi r^2 + \frac{2V_0}{r} = \infty + 0$$

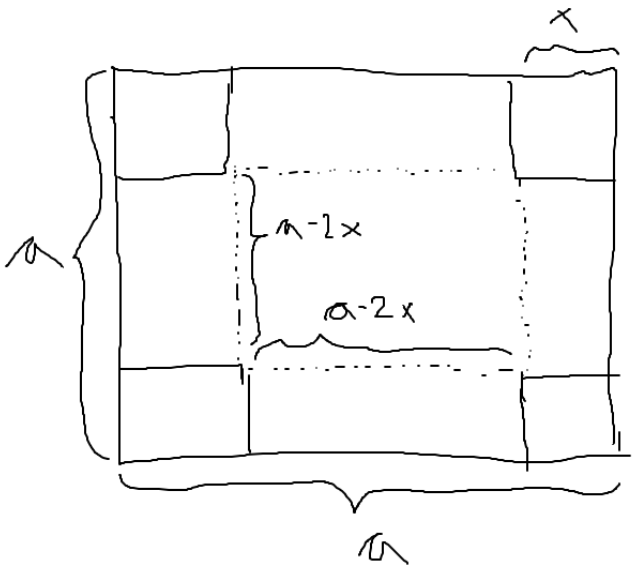


Torej ima optimalna posoda obliko $r = \sqrt[3]{\frac{V_0}{\pi}}$ in $h = \frac{V_0}{\pi \sqrt[3]{\frac{V_0}{\pi}}^{\,2}}$

$$= \frac{V_0^{1/3}}{\pi^{1/3}}$$

$$= \sqrt[3]{V_0/\pi}$$

4. Iz vogalov kvadrata s stranico a izrežemo 4 vogalne kvadrate in jih vržemo stran. Iz preostanka sestavimo škatlo brez pokrova. Kako naj izrežemo enake vogalne kvadratke, da bo škatla imela največji volumen?

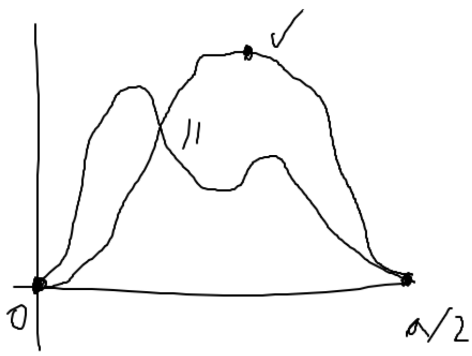


$$V = P_{\text{osnovne ploskve}} \cdot h$$
$$= (a-2x)^2 \cdot x =: f(x)$$

maksimiziramo f na $\left[0, \frac{a}{2}\right]$

$$f = (2x-a)^2 \cdot x = (4x^2-4ax+a^2)x = 4x^3-4ax^2+a^2x$$

$$f(0) = 0$$
$$f\left(\tfrac{a}{2}\right) = \left(\cancel{2\cdot\tfrac{a}{2} - a}\right)^2 \cdot \tfrac{a}{2} = 0$$
$$f' = 12x^2 - 8ax + a^2$$



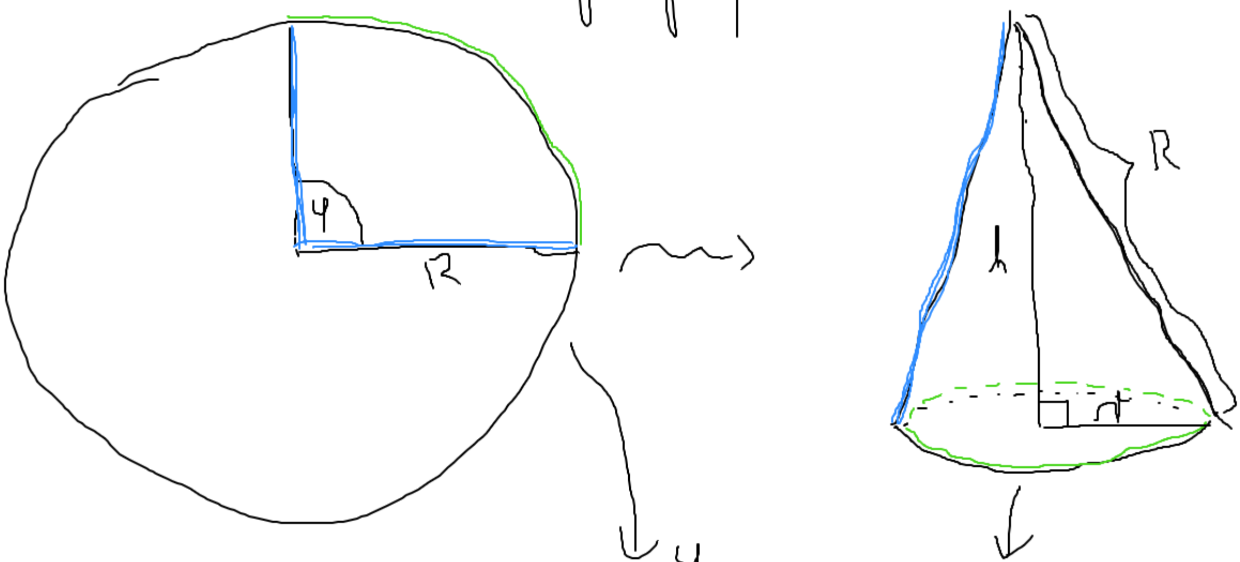
stacionarne točke:

$$f'(x) = 0 \iff 12x^2 - 8ax + a^2 = 0$$
$$\iff x_{1,2} = \frac{8a \pm \sqrt{64a^2 - 4\cdot 12\cdot a^2}}{2\cdot 12} = \frac{8a \pm \sqrt{16a^2}}{24} = \frac{(8\pm4)a}{24} = \begin{cases} \frac{a}{2} \\ \frac{a}{6} \end{cases}$$

$\implies x = \frac{a}{6} \implies$ dimenzije škatle: $a - 2x = \frac{4a}{6}$ ... osnovna ploskev

$\frac{a}{6}$ ... višina

5 Iz kroga z danim polmerom R izrežemo izsek in ga zvijemo v stožec. Pri katerem kotu izseka bo imel stožec največjo prostornino?



$\varphi \in [0, 2\pi]$, obseg $= \cancel{2\pi} R \cdot \frac{\varphi}{\cancel{2\pi}} = \varphi R = 2\pi r$

$$\Rightarrow r = \frac{\varphi R}{2\pi}$$

$$h = \sqrt{R^2 - r^2} = \sqrt{R^2 - \frac{\varphi^2 R^2}{(2\pi)^2}} = \sqrt{\frac{R^2(4\pi^2 - \varphi^2)}{(2\pi)^2}} = \frac{R\sqrt{4\pi^2 - \varphi^2}}{2\pi}$$

volumen stožca $= P_{\text{osnovne ploskve}} \cdot h \cdot \frac{1}{3} = \pi r^2 \cdot \frac{h}{3} =$

$$= \frac{\pi}{3}\left(\frac{\varphi R}{2\pi}\right)^2 \cdot \frac{R\sqrt{4\pi^2 - \varphi^2}}{2\pi} = \frac{R^3 \varphi^2 (4\pi^2 - \varphi^2)^{1/2}}{\pi^2 \cdot 24} =: f(\varphi), \quad \varphi \in [0, 2\pi]$$

maksimiziramo f

$f(0) = 0$, $f(2\pi) = 0$

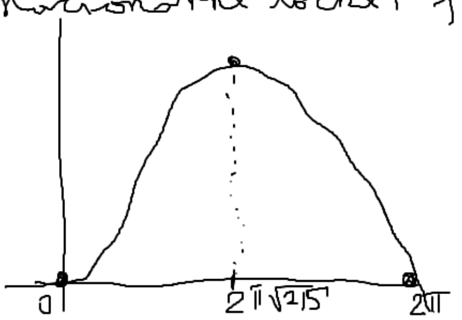
$$f' = \frac{R^3}{\pi^2 24} \cdot \left(2\varphi\sqrt{} + \varphi^2 \frac{1}{2}()^{-1/2} \cdot (-2\varphi)\right) = \frac{R^3}{24\pi^2}\left(2\varphi\sqrt{4\pi^2 - \varphi^2} - \frac{\varphi^3}{\sqrt{4\pi^2 - \varphi^2}}\right)$$

$$= \frac{R^3 \left(2\varphi(4\pi^2 - \varphi^2) - \varphi^3\right)}{24\pi^2 \sqrt{4\pi^2 - \varphi^2}} = \frac{R^3 (8\pi^2\varphi - 3\varphi^3)}{24\pi^2\sqrt{4\pi^2 - \varphi^2}}$$

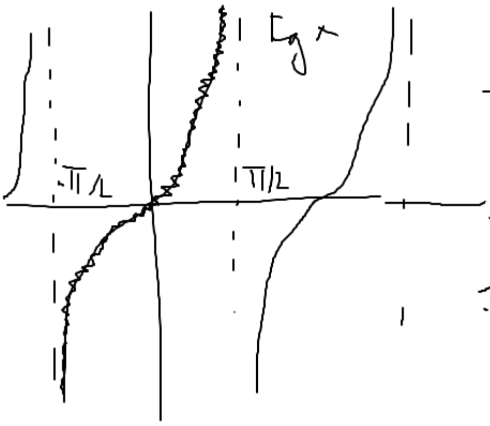
stacionarne točke: $f' = 0 \Leftrightarrow 8\pi^2\varphi - 3\varphi^3 = 0 \Leftrightarrow \varphi(8\pi^2 - 3\varphi^2) = 0$

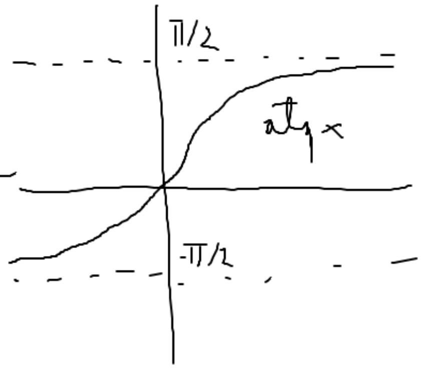
$\varphi = 0$ ali $\varphi = \sqrt{\frac{8\pi^2}{3}}$

$$= \underline{\underline{2\pi\sqrt{2/3}}}$$



G • $$\lim_{x \to \infty} x(\pi - 2\,\mathrm{atg}\,x) \overset{\infty \cdot (\pi - \pi)}{=} \lim_{x \to \infty} \frac{\pi - 2\,\mathrm{atg}\,x}{x^{-1}} \overset{\text{L'H}}{=} \lim_{x \to \infty} \frac{-2\frac{1}{1+x^2}}{-x^{-2}} \begin{matrix} \cdot x^2 \\ \cdot x^2 \end{matrix}$$





$$= \lim_{x \to \infty} \frac{+2x^2}{1+x^2} = \underline{\underline{2}}$$