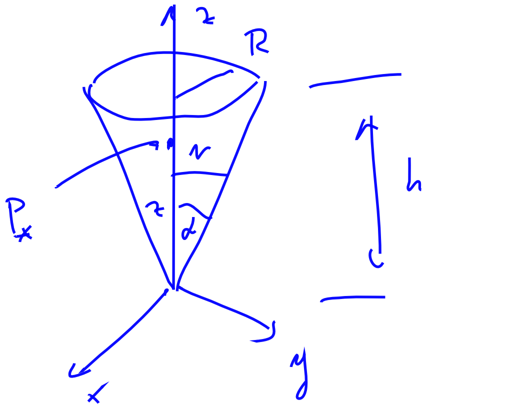

Vaje 28. december 2021

$\operatorname{tg}\alpha = \frac{R}{h} = \frac{r}{z}$

$r = \frac{R}{h} z$

homogeni

1. Izračunaj vztrajnostni tenzor stožca:
   (a) s polom v vrhu stožca;
   (b) s polom v masnem središču;
   (c) v koordinatnem sistemu z eno osjo v smeri tvorilke stožca.

$$K_x = \int_B x^2 dm = \rho_0 \int_B x^2 dV = \rho_0 \int_0^h dz \int_0^{2\pi} d\varphi \int_0^{Rz/h} r\,dr\, r^2 \cos^2\varphi =$$

$$= \rho_0 \int_0^h dz \int_0^{2\pi} \cos^2\varphi\, d\varphi \int_0^{Rz/h} r^3 dr = \rho_0 \int_0^h dz\, \pi \frac{1}{4} \frac{R^4}{h^4} z^4 = \frac{1}{4}\rho_0 \frac{R^4}{h^4}\pi \frac{1}{5} h^5 =$$

$$= \frac{1}{20}\rho_0 R^4 h \pi \qquad m = \rho_0 V$$

$$V = \int_B dV = \int_0^h dz \int_0^{2\pi} d\varphi \int_0^{Rz/h} r\,dr = 2\pi \int_0^h \frac{1}{2}\frac{R^2 z^2}{h^2} dz = \pi \frac{R^2}{h^2}\frac{1}{3}h^3 = \frac{1}{3}\pi R^2 h$$

$$\underline{K_x = \frac{3}{20} m R^2} \qquad K_y = \int_B y^2 dm = \rho_0 \int_0^h dz \int_0^{2\pi} d\varphi \int_0^{Rz/h} r\,dr\, r^2 \sin^2\varphi = \underline{\frac{3}{20} m R^2}$$

$$K_z = \int_B z^2 dm = \rho_0 \int_0^h dz \int_0^{2\pi} d\varphi \int_0^{Rz/h} r\,dr\, z^2 = \rho_0 \cdot 2\pi \int_0^h dz \cdot \frac{1}{2}\frac{R^2 z^2}{h^2}\cdot z^2 =$$

$$= \rho_0 \pi \frac{R^2}{h^2} \int_0^h z^4 dz = \frac{1}{5}\rho_0 \pi R^2 h^3 = \frac{3}{5} m h^2$$

$$J_1 = K_y + K_z = \frac{3}{5} m \underline{\left(\frac{1}{4}R^2 + h^2\right)} = J_2$$

$$J_3 = K_x + K_y = \underline{\frac{3}{10} m R^2}$$

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b) $\underline{J}(P_0) = \underline{J}(P_*) + m|P_0 - P_*|^2 \underline{I} - m(P_0 - P_*)\otimes(P_0 - P_*)$

določitev P_*

$$z_* = \frac{1}{m}\int z\,dm = \frac{1}{V}\int z\,dV = \frac{3}{\pi R^2 h}\cdot\frac{1}{4}\pi R^2 h^2 = \frac{3}{4}h$$

$$\int_B z\,dV = \int_0^h dz\int_0^{2\pi} d\varphi \int_0^{Rz/h} r\,dr\cdot z = 2\pi\,\frac{1}{2}\int_0^h \frac{R^2}{h^2}z^3\,dz = \pi\frac{R^2}{h^2}\cdot\frac{1}{4}h^4 = \frac{1}{4}\pi R^2 h^2$$

$$\underline{\underline{J}}(P_0) = \underline{\underline{J}}(P_*) + m\frac{9}{16}h^2\,\underline{\underline{I}} - m\frac{9}{16}h^2\,\vec{k}\otimes\vec{k}$$

$$\underline{\underline{I}} = \begin{bmatrix}1&&\\&1&\\&&1\end{bmatrix} = \vec{i}\otimes\vec{i} + \vec{j}\otimes\vec{j} + \vec{k}\otimes\vec{k}$$

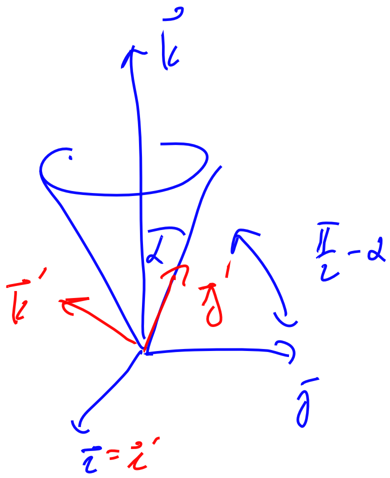
$$\underline{\underline{J}}(P_0) = \underline{\underline{J}}(P_*) + m\frac{9}{16}h^2(\vec{i}\otimes\vec{i} + \vec{j}\otimes\vec{j})$$

$$\frac{3}{5}m\left(\frac{1}{4}R^2 + h^2\right)(\vec{i}\otimes\vec{i} + \vec{j}\otimes\vec{j}) + \frac{3}{10}mR^2\,\vec{k}\otimes\vec{k}$$

$$\frac{3}{5} + \frac{9}{16} = \frac{48+45}{80} = \frac{93}{80}$$

$$= \frac{3}{5}m\begin{bmatrix}\frac{1}{4}R^2 + \frac{93}{80}h^2 & & \\ & \frac{1}{4}R^2 + \frac{93}{80}h^2 & \\ & & \frac{1}{2}R^2\end{bmatrix}$$

c) $J'_{ij} = \alpha_{ik} J_{kl}\,\alpha_{jl}$



$$\vec{e}_i{}' = \alpha_{ik}\vec{e}_k$$

$\vec{e}_1{}' = \vec{e}_1 \Rightarrow \alpha_{11} = 1,\quad \alpha_{12} = \alpha_{13} = 0$

$\vec{e}_2{}' = \sin\alpha\,\vec{e}_2 + \cos\alpha\,\vec{e}_3 \Rightarrow \alpha_{21} = 0,\ \alpha_{22} = \sin\alpha,\ \alpha_{23} = \cos\alpha$

$\vec{e}_3{}' = -\cos\alpha\,\vec{e}_2 + \sin\alpha\,\vec{e}_3 \Rightarrow \alpha_{31} = 0,\ \alpha_{32} = -\cos\alpha,\ \alpha_{33} = \sin\alpha$

$$J'_{11} = \alpha_{1k}J_{kl}\alpha_{1l} = \alpha_{11}J_{11}\alpha_{11} = J_{11}$$

$$J'_{12} = \alpha_{1k}J_{kl}\alpha_{2l} = \alpha_{11}J_{1l}\alpha_{2l} = J_{11}\underset{}{\overset{0}{\alpha_{21}}} + \overset{0}{J_{12}}\alpha_{22} + \overset{0}{J_{13}}\alpha_{23} = 0$$

$$J'_{13} = \alpha_{1k}J_{kl}\alpha_{3l} = J_{1l}\alpha_{3l} = 0$$

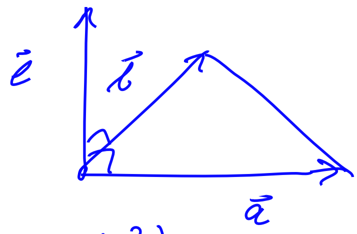
$$J'_{22} = \alpha_{2k} J_{kl} \alpha_{2l} = \sum_{k=1}^{3} \alpha_{2k} J_{kk} \alpha_{2k} = \sin^2\alpha \, J_{22} + \cos^2 J_{33}$$

$$J'_{23} = \alpha_{2k} J_{kl} \alpha_{3l} = \sum_{k=1}^{3} \alpha_{2k} J_{kk} \alpha_{3k} = -\sin\alpha \cos\alpha \, J_{22} + \cos\alpha \sin\alpha \, J_{33}$$

$$J'_{33} = \alpha_{3k} J_{kl} \alpha_{3l} = \sum_{k=1}^{3} \alpha_{3k}^2 J_{kk} = \cos^2\alpha \, J_{22} + \sin^2\alpha \, J_{33}$$

$$\begin{bmatrix} J'_{11} & 0 & 0 \\ 0 & J'_{22} & J'_{23} \\ 0 & J'_{23} & J'_{33} \end{bmatrix}$$

2. Izračunaj vztrajnostni moment trikotne plošče okrog osi pravokotne na ravnino trikotnika.



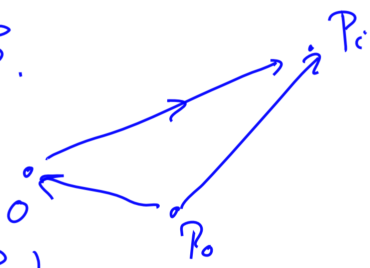
$$J = \frac{1}{6} m \left(|\vec{a}|^2 + \vec{a} \cdot \vec{b} + |\vec{b}|^2 \right)$$

3. Ugotovi kdaj ima sistem sil $\mathcal{F}$ skupno prijemališče.

$\mathcal{F} = \{(P_1, \vec F_1), \dots, (P_n, \vec F_n)\}$ $\qquad \vec F(\mathcal F) = \sum_{i=1}^n \vec F_i$

$$\vec N(\mathcal F, O) = \sum_{i=1}^n (P_i - O)\times \vec F_i$$

Sistem sil ima skupno prijemališče v točki P_0, če je $\vec N(\mathcal F, P_0) = \vec 0$.



$$\vec N(\mathcal F, P_0) = \sum_{i=1}^n (P_i - P_0)\times \vec F_i =$$

$$P_i - O = (P_i - O) + (O - P_0)$$

$$= \vec N(\mathcal F, O) + (O-P_0)\times \vec F(\mathcal F)$$

Iščem P_0 takšen, da je $\vec 0 = \vec N(\mathcal F, O) + (O-P_0)\times \vec F(\mathcal F) \quad / \times \vec F(\mathcal F)$

$$\vec 0 = \vec N(O)\times\vec F + ((O-P_0)\times \vec F)\times \vec F =$$

$$= \vec N(O)\times \vec F + ((O-P_0)\cdot\vec F)\vec F - (\vec F\cdot\vec F)(O-P_0) \qquad (O-P_0)\cdot \vec F = 0$$

$$\vec 0 = \vec N(O)\times\vec F - |\vec F|^2(O-P_0) \Rightarrow O - P_0 = \frac{\vec N(O)\times \vec F}{|\vec F|^2}$$

$$\boxed{P_0 - O = \frac{\vec F\times\vec N(O)}{|\vec F|^2}} + \lambda \vec F$$

$$\vec N(P_0) = \vec N(\mathcal F, O) + (O-P_0)\times \vec F(\mathcal F) = \qquad \text{"} \vec c \text{"}$$

$$= \vec N(\mathcal F, O) - \frac{1}{|\vec F|^2}(\vec F\times \vec N(O))\times \vec F - \lambda \vec F\times\vec F$$

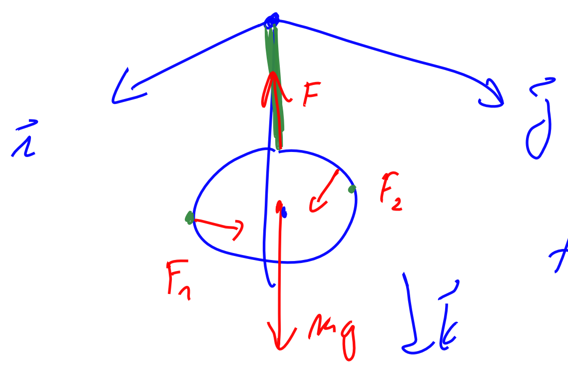
$$= \vec N(\mathcal F, O) - \frac{\vec N(O)(\vec F\cdot\vec F)}{|\vec F|^2} + \frac{\vec F\cdot(\vec N(O)\cdot\vec F)}{|\vec F|^2} = \frac{\vec F}{|\vec F|^2}(\vec N(O)\cdot\vec F)$$

Skupno prijemališče obstaja, če je $\boxed{\vec N(\mathcal F, O)\cdot \vec F(\mathcal F) = 0}$

$\vec{I} = \vec{N}(\mathcal{F}, O) \cdot \vec{F}(\mathcal{F})$ to je invarianta sistema sil, je neodvisna od pola O.

$$\vec{N}(\mathcal{F}, P_0) = \vec{N}(\mathcal{F}, O) + (O - P_0) \times \vec{F}(\mathcal{F}) \qquad \vec{N}(\mathcal{F}, P_0)\cdot\vec{F}(\mathcal{F}) = \vec{N}(\mathcal{F}, O)\cdot\vec{F}(\mathcal{F})$$

4. Določi silo vrvice na kroglo v vogalu med dvema pravokotnima stenama. .



$m\vec{a}_* = \vec{F}$

$\frac{d\vec{\mathcal{L}}(P_*)}{dt} = \vec{N}(P_*)$

$\vec{F} = \vec{0}$

$\vec{N}(P_*) = \vec{0}$

+

mirovanje v $t=0$

tedaj sledi mirovanje $\Leftarrow$

Sistem sil $\mathcal{F}$ je ravnovesni sistem, če je $\vec{F}(\mathcal{F}) = \vec{0}$ in

$\vec{N}(\mathcal{F}, O) = \vec{0}$

$$\vec{N}(\mathcal{F}, P_0) = \vec{N}(\mathcal{F}, O) + (O - P_0) \times \vec{F}(\mathcal{F})$$

Ravnovesne enačbe so: $\vec{F} + \vec{F}_1 + \vec{F}_2 + m\vec{g} = \vec{0}$

$\vec{N}(\mathcal{F}, O) = \vec{0}$

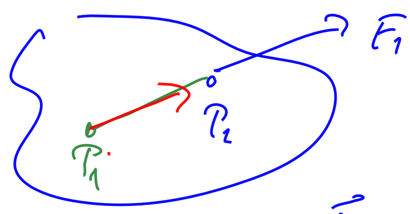
V primeru, da ima sistem sil skupno prijemališče P_0 je momentna enačba trivialna $\vec{0} = \vec{0}$

$$\vec{N}(\mathcal{F}, P_1) = \vec{0}$$

Princip polzmiti $\quad (P_1, \vec{F}_1) \equiv (P_1 + \lambda \vec{F}_1, \vec{F}_1)$

$F_2 + \frac{r}{l+r} F = 0$ //

$F \frac{z^*}{l+r} + mg = 0$

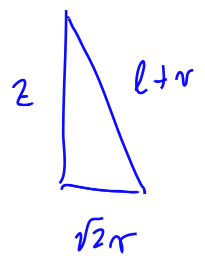


$$((P_1 + \lambda \vec{F}_1) - O) \times \vec{F}_1 = ((P_1 - O) + \lambda \vec{F}_1) \times \vec{F}_1 = (P_1 - O) \times \vec{F}_1$$

$\vec{F}_1 = F_1 \vec{i}, \quad \vec{F}_2 = F_2 \vec{j}, \quad m\vec{g} = mg\vec{k}$

$$\vec{F} = \frac{(O - P_*)}{|O - P_*|} F = \frac{(O - P_*)}{l+r} F \qquad F = \frac{1}{l+r}(r\vec{i} + r\vec{j} + z_*\vec{k})$$

$$\boxed{F = -\frac{mg(l+r)}{z^*}}$$



$P_*(r, r, z_*)$

$F_1 + \frac{r}{l+r} F = 0$ //

$(l+r)^2 = z^2 + 2r^2$

$z_* = \sqrt{(l+r)^2 - 2r^2} = \sqrt{l^2 + 2rl - r^2}$