Geometry (part 2)
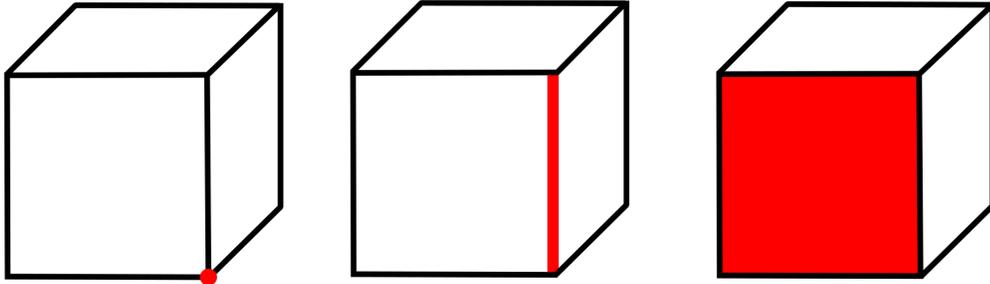
1. Fill in the blanks

   a) Two triangles that have the same shape and size are called _____ triangles.

   b) Two triangles that have the same shape but not necessarily the same size are called _____ triangles.

   c) Two triangles are congruent if and only if _____.

   d) Two triangles are similar if and only if _____.

2. Name each of the following.

3. Last time we defined regular polygon as a polygon in which all sides have the same length and all interior angles have the same size. We can do something similar for polyhedra. Fill in the blanks.

   a) In regular polyhedron all _____ are all the same kind of regular polygon, and the same number of faces meet at every _____. Polyhedra with these two properties are called _____.

   b) If we create a polyhedron where three equilateral triangles meet at every vertex, we get _____. It has _____ faces.

   c) If four equilateral triangles meet at every vertex, we get a different Platonic solid. It is called the _____ and has _____ faces.

   d) If _____ triangles meet at every vertex, we get the Icosahedron. It has _____ faces.

e) If _____ triangles meet at every vertex, something different happens: we simply get a _____, instead of a three-dimensional polyhedron.

f) And _____ triangles at every vertex also don't produce new polyhedra: there is not enough space around a vertex, to fit that many triangles. This means we've found _____ Platonic solids consisting of triangles.

g) If _____ squares meet at every vertex, we get the cube. Just like dice, it has _____ faces. The cube is sometimes also called _____.

h) If 4 squares meet at every vertex, we get a _____. And like before, five or more squares also won't work.

i) If _____ pentagons meet at every vertex, we get the Dodecahedron. It has _____ faces.

j) Like before, four or more pentagons (don't work/are possible) _____ because there is not enough space.

k) The next regular polygon to try are hexagons. If three hexagons meet at every vertex, we immediately get a _____. Since there is no space for more than three, it seems like there are no Platonic solids consisting of hexagons.

4. From the previous exercise we deduce that there are just _____ Platonic solids! Let's have a look at all of them together:
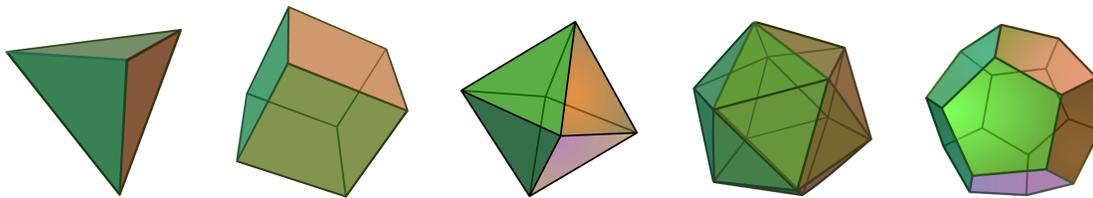
a) Count the number of vertices V, edges E and faces F of each of the five Platonic solids.

b) Notice how the number of faces and vertices are swapped around for cube and octahedron, as well as dodecahedron and icosahedron, while the number of edges stays the same. These pairs of Platonic solids are called dual solids. We can turn a polyhedron into its dual, by "replacing" every face with a vertex, and every vertex with a face.

c) Calculate $V - E + F$ for each.

5. Count the number of vertices V, edges E and faces F of pyramid, prism and Buckminsterfullerene (figure is on the next page). Then compute the value $V - E + F$.
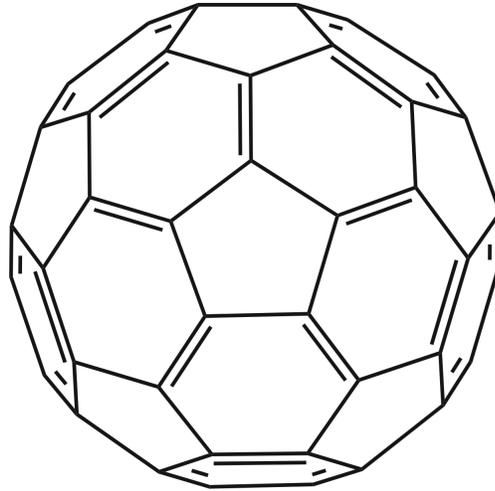
The number $V - E + F$ is called Euler characteristic and is denoted by χ. Any convex polyhedron has Euler characteristic $V - E + F = 2$.

6. Derive a formula for the surface area and volume of a cylinder. What about the surface area of a cone?