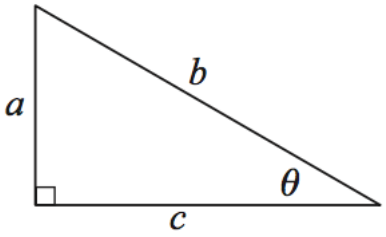
Trigonometry

1. The remaining angle of the illustrated triangle is $90° - \theta$ which is the complement of θ (recall that two angles are *complementary* if their sum is $90°$).



   a) In terms of a, b and c, find

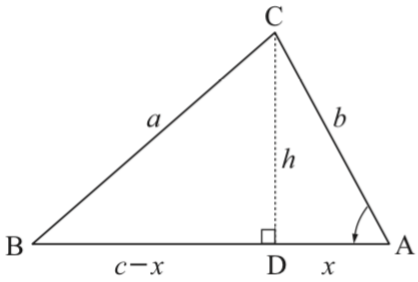
      i) $\sin\theta$ ii) $\cos\theta$ iii) $\sin(90° - \theta)$ iv) $\cos(90° - \theta)$

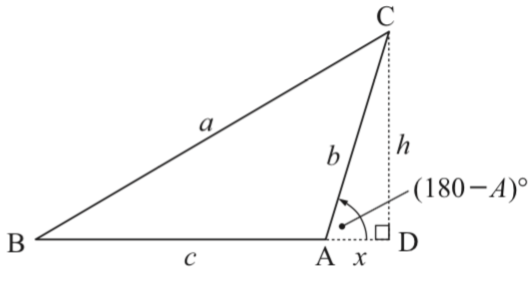
   b) Use your results from (a) to complete the following statements:

      i) The sine of an angle is the cosine of its ...

      ii) The cosine of an angle is the sine of its ...

2. In this exercise we will prove a *generalisation* of Pythagoras' theorem for triangles which need not be right-angled.





   For each triangle above

   a) Write a^2 in terms of c, x and h.

   b) Write h^2 in terms of x and b.

c) Find a formula relating $\cos A$, x and b.

d) Hence deduce that $a^2 = b^2 + c^2 - 2bc\cos A$.

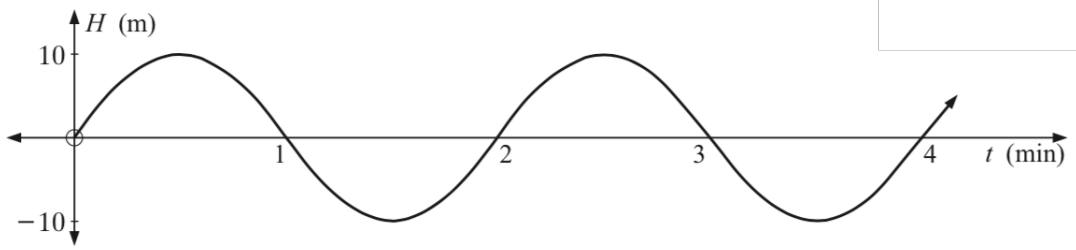
Summary: For any triangle ΔABC, whether acute, obtuse or right-angled, we have the formula $a^2 = b^2 + c^2 - 2bc\cos A$. This is known as the *cosine rule*.

3. Convert the following from rectangular (also known as Cartesian) coordinates to polar coordinates, and vice versa.

a) The point $(1, \sqrt{3})$, b) The point $(1, 0)$,

c) The point with $r = 10$, $\theta = \frac{\pi}{6}$, b) The point with $r = 5$, $\theta = \frac{\pi}{4}$.

4. A particular seat on a Ferris wheel has a height function $h(t) = 10\sin(\pi t)$.



a) From the graph, how do we determine the diameter of the wheel?

b) How long does it take the wheel to complete one revolution?

c) Suppose the operator decides to slow the Ferris wheel to half its current speed. What is the new height function?

5. How many radians are there in:

a) 45° b) 60° c) 90° d) 180° e) 270° f) 360°?

6. Without using a calculator, work out:

a) $\cos(\frac{\pi}{4})$, b) $\sin(\frac{\pi}{4})$, c) $\cos(\frac{\pi}{6})$

Hint: Consider the right-angled triangle with both legs of length 1, and the equilateral triangle with all sides of length 1.

7. Prove the following:

$$\sin(\arctan(x)) = \frac{x}{\sqrt{1+x^2}}, \qquad \cos(\arctan(x)) = \frac{1}{\sqrt{1+x^2}}.$$

Hint: Consider the right-angled triangle with legs of length 1 and x.

8. a) Prove the formula

$$\cos\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1+\cos(x)}{2}}.$$

Hint: Use the identities

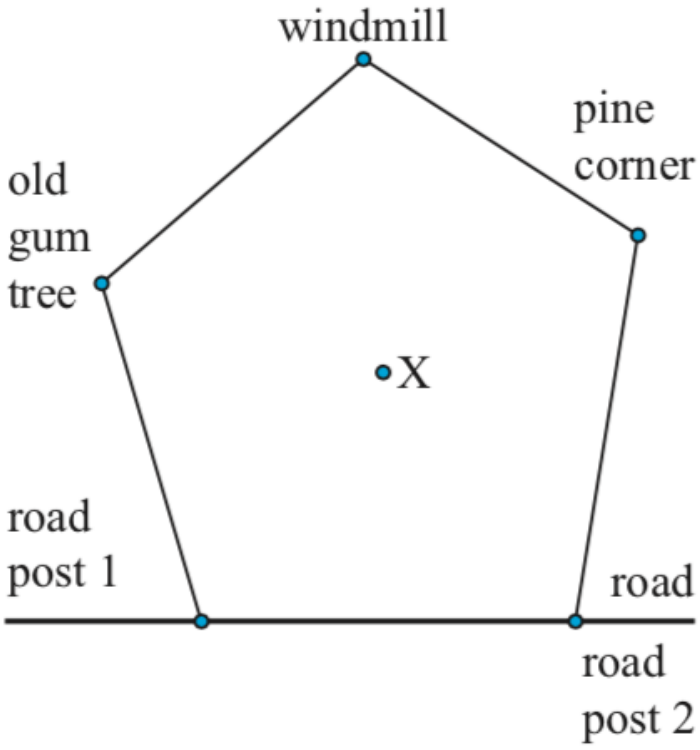
$$\cos^2 x + \sin^2 x = 1, \qquad \cos(2x) = \cos^2 x - \sin^2 x$$

b) Prove that

$$\cos\left(\frac{\pi}{12}\right) = \frac{1+\sqrt{3}}{2\sqrt{2}}.$$

9. Anja has her property surveyed. The surveyor chooses a point X within the property and using their laser device measures the distances of the boundary corners from X. The surveyor also gives bearings of the corner points from X.

pine corner	37 m	048
road post 2	44 m	132
road post 1	41 m	213
old gum tree	52 m	280
windmill	51 m	343



Use this information to find:

a) the area of the property,

b) the perimeter of the property.