Matrices

1. A builder builds a block of 12 identical flats. Each flat is to contain 1 table, 4 chairs, 2 beds and 1 wardrobe. If $v = \begin{bmatrix} 1 \\ 4 \\ 2 \\ 1 \end{bmatrix}$ is the matrix representing the furniture in one flat, what, in terms of v, is the matrix representing the furniture in all flats?

2. In the school swimming competition, a first place is awarded 10 points, second place 6 points, third place 3 points and fourth place 1 point. One school won 3 first places, 2 seconds, 4 thirds and 2 fourths.

   a) Write down this information in terms of a points vector P and a numbers vector N.

   b) Show how to use P and N to determine the total number of points awarded to the school.

3. For matrices $A = \begin{bmatrix} 1 & 2 & 0 \\ 3 & 1 & -1 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 2 & -2 \\ 0 & 1 & 2 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 3 & 1 \\ 1 & -1 & 1 \end{bmatrix}$ perform (if it is possible) the following operations: $A + B$, $3A - 2B$, $2A + 3C$, C^2, AC, CA, A^T.

4. Prove that $(AB)^T = B^T A^T$ holds for all $A \in \mathbb{R}^{m \times n}$ and $B \in \mathbb{R}^{n \times p}$.

5. a) We say that matrices A and B _____ if $AB = BA$.

    b) If $A(BC) = (AB)C$ holds, we say that the matrix multiplication is _____.

    c) We say that _____ property of matrices holds if $A(B + C) = AB + AC$ and $(A + B)C = AC + BC$.

6. In November, Lou E Gee sold 23 fridges, 17 stoves and 31 microwave ovens and his partner Rose A Lee sold 19 fridges, 29 stoves and 24 microwave ovens. In December Lou's sales were: 18 fridges, 7 stoves and 36 microwaves while Rose's sales were: 25 fridges, 13 stoves and 19 microwaves.

   a) Write their sales for November as a 3×2 matrix.

   b) Write their sales for December as a 3×2 matrix.

   c) Write their total sales for November and December as a 3×2 matrix.

7. A grocer at the market sells 6 boxes of apples, 7 boxes of bananas and 9 boxes of oranges. The next day, 5 boxes of apples, 8 boxes of bananas and 4 boxes of oranges are sold. On the third day, 4 boxes of apples, 7 of bananas and 2 of oranges are sold. The apples cost 18€ a box, bananas 15€ a box and oranges 13€ a box over the 3-day period. Express this information in the form of two matrices and show how to use the matrices to find the total cost of the fruit.

8. Consider the matrix
$$R = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}.$$

   a) For $\theta_1 = \frac{\pi}{3}$ and $\theta_2 = \frac{\pi}{2}$ compute the products Rv_1 and Rv_2, where $v_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $v_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

   b) Can you deduce what the matrix represents? It represents an anticlockwise rotation about the origin by an angle θ.

   c) Intuitively, we know that a rotation by $\frac{\pi}{3}$ followed by a rotation by $\frac{\pi}{6}$, and a rotation by $\frac{\pi}{6}$ followed by a rotation by $\frac{\pi}{3}$ in the same direction, should both result in the same thing. Can we demonstrate this using the rotation matrices? What does this matrix represent?

   d) What matrix do you get if you first rotate by θ anticlockwise and then you rotate by the same angle in the opposite direction?