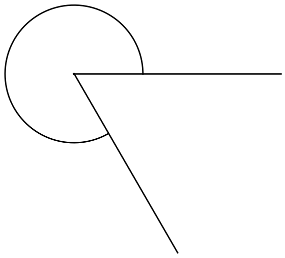
Expressing Mathematics in English - First Midterm Test

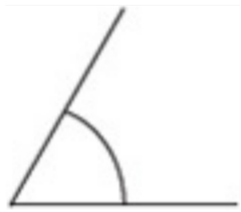
22 February 2021

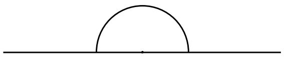
You have 90 minutes to answer the following six questions. Please write neatly, and please ensure that you follow all instructions on the "Information about the midterm test" *page on the course webpage. Before submitting, remember to check that all pages of your answer show your name and student id, and are numbered.*

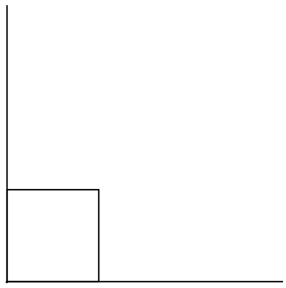
(1) Choose the appropriate name for the type of angle in each picture from the list below.

(a) 

(b) 

(c) 

(d) 

(e) 

right, complementary, abstruse, straight, obtuse, reflex, regular, acute

(2)(a) Define in words what it means for a triangle to be isosceles, and illustrate your definition with a sketch.

(b) If a triangle has a base of length 10 centimetres and two other sides each of length 13 centimetres, what is its height? Give the names of any theorems you use to calculate the answer.

(c) What is the area of the triangle described in (b), in square metres? Please give your answer in scientific notation.

(3)(a) Explain in English how to simplify a fraction to its reduced form.

(b) Illustrate your answer, by simplifying the fraction $\frac{242}{77}$.
(You should explain all steps taken, as well as giving the final answer.)

(4) Write down the words that appropriately fill in the blanks in the following paragraphs. Some of the blanked words appear more than once, in which case they are given the same letter (this may help you work out what they are); you do not have to write them more than once.

- The ___(a)___ of exponentiation is defined as e^x, where e is Euler's ___(b)___, which has a ___(c)___ of ___(d)___ 2.718. The ___(e)___ ___(a)___ to exponentiation is called the ___(f)___ ___(g)___ and written $\ln(x)$. More generally the ___(e)___ ___(a)___ to the function b^x is called the ___(h)___ b ___(g)___ of x. Logarithms have the ___(i)___ that the ___(g)___ of a product of numbers is the ___(j)___ of the logarithms of the individual numbers.

- There are five regular ___(k)___, which are collectively known as the ___(l)___ solids. A (regular) ___(m)___ has four ___(n)___; a ___(o)___ has six ___(n)___; an ___(p)___has eight ___(n)___; a ___(q)___ has twelve ___(n)___; and an icosahedron has ___(r)___ ___(n)___. Regular ___(k)___ have the property that every face is a regular ___(s)___ and that the same ___(t)___ of ___(n)___ meet at every vertex of the polyhedron.

(5) Translate the following sentences into English.

(a) Kvadratna matrika je matrika, ki ima enako število stolpcev in vrstic.

(b) Množenje dveh matrik je izvedljivo le, če je število stolpcev prve matrike enako številu vrstic druge matrike.

(c) Za vsako matriko je njen rang po vrsticah enak rangu po stolpcih.

(6) Read the text on the following page, and answer the multiple choice questions that follow.

Consider any finite list of prime numbers $p_1, p_2, \ldots, p_n$. It will be shown that there exists at least one additional prime number that is not in this list. Let P be the product of all the prime numbers in the list; i.e., $P = p_1 p_2 \ldots p_n$. Let $q = P + 1$. Then q is either prime or not.

- If q is prime, then it is a prime number that is not in the list.
- If q is not prime, then some prime factor p divides q. If this factor p were in our list, then it would divide P (since P is the product of every number in the list); but p also divides $q = P + 1$, as just stated. If p divides both P and q, then p must also divide the difference of the two numbers, which is $(P+1) - P$ or just 1. But no prime number divides 1. Thus the prime p cannot be on our list. This means that at least one more prime number exists beyond those in the list.

(a) Which of the following statements is proved by the argument above?
 (A) It is impossible to write finitely many prime numbers in a list.
 (B) The uniqueness of prime factorisation.
 (C) There are infinitely many prime numbers.
 (D) The product of finitely many prime numbers is prime.

(b) The abbreviation "*i.e.*" in the third line means:
 (A) "if ever"
 (B) "increasingly enumerated"
 (C) "that is"
 (D) "as an equality"

(c) Why is the statement "q is either prime or not" true?
 (A) By the laws of logic.
 (B) It isn't true, this is a proof by contradiction.
 (C) Only because q is $P + 1$ where P is a product of primes.
 (D) Because if it is true, it follows that $P - 1$ is also prime.

(d) Why is the number q not in the list?
 (A) Because p_n is the largest number in the list.
 (B) Because $q > p_i$ for every $i = 1 \ldots n$.
 (C) Because there are infinitely many prime numbers.
 (D) Because only prime numbers are in the list.

(e) Why does some prime factor p divide q, if q is not prime?
 (A) By the uniqueness of prime factorisation.
 (B) Because every number greater than 2 that is not prime has a factor that is prime.
 (C) Because every number that is not prime has a factor that is prime.
 (D) Because p can be chosen from the list $p_1, p_2, \ldots, p_n$.

(f) Why must p also divide the difference of q and P?
 (A) Because every common divisor of x and y divides $x - y$.
 (B) Because every number has a prime factorisation.
 (C) Because p is the greatest common divisor of P and q.
 (D) Because $q - P$ is not a prime number.

(g) Why does no prime number divide 1?
 (A) Because prime numbers don't divide numbers different from themselves.
 (B) Because the only numbers that divide a prime are 1 and the number itself.
 (C) Because 1 is a prime number.
 (D) Because 1 is not a prime number.

(h) What is the argument justifying the statement that "p cannot be on our list"?
 (A) Having assumed that p is on the list, we have reached a contradiction.
 (B) Because no prime number divides 1.
 (C) Because at least one more prime number exists beyond those in the list.
 (D) We did not write p in our list of numbers $p_1, p_2, \ldots, p_n$.