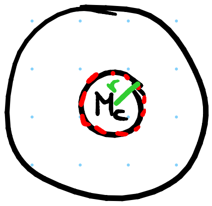
Privzemimo, da ima zvezda uniformno neprozornost κ in uniformen β ter da je jedro konvektivno. Jedrske reakcije so edini izvor energije, ki torej nastaja samo v jedru. Pokaži, da je masni delež jedra enak

$$\frac{\gamma_a}{4(\gamma_a-1)}\,.$$



M_c – masa konvektivnega jedra zvezde

$$\left.\begin{aligned}\frac{dT}{dr} &= \left(\frac{\gamma_a-1}{\gamma_a}\right)\frac{T}{p}\frac{dp}{dr} \\ &\overset{!}{=} -\left(\frac{\gamma_a-1}{\gamma_a}\right)\frac{T}{p}\frac{GM_c\rho}{r^2}\end{aligned}\right\}\ \text{konvektivno jedro}$$

$$\left.\frac{dT}{dr} = -\frac{3}{4ac}\frac{\kappa\rho}{T^3}\frac{L(r)}{4\pi r^2}\right\}\ \text{ovojnica}$$

$$\cancel{-}\left(\frac{\gamma_a-1}{\gamma_a}\right)\frac{T}{p}\frac{GM_c\rho}{r^2} = \cancel{-}\frac{3}{4ac}\frac{\kappa\rho}{T^3}\frac{L(r)}{4\pi r^2}$$

$$L(r) = L(R) \qquad \frac{\frac{1}{3}aT^4}{p} = 1-\beta$$

$$\left(\frac{\gamma_a-1}{\gamma_a}\right)(1-\beta)\frac{GM_c\cancel{\rho}}{\cancel{r^2}} = \frac{1}{4c}\frac{\kappa\cancel{\rho}\,L(R)}{4\pi\cancel{r^2}}$$

$$M_c = \frac{\gamma_a}{(\gamma_a-1)(1-\beta)}\cdot\frac{1}{4}\cdot\frac{\kappa L(R)}{4\pi c G} \qquad \Big|:M$$

$$\frac{M_c}{M} = \frac{\gamma_a}{(\gamma_a-1)(1-\beta)4}\cdot\frac{\kappa L}{4\pi cGM} \longrightarrow \frac{4\pi cGM}{\kappa} = L_{Edd}$$

$$\frac{L}{L_{Edd}} = 1-\beta$$

$$\frac{M_c}{M} = \frac{\gamma_a}{(\gamma_a-1)\cancel{(1-\beta)}4}\cdot\cancel{(1-\beta)}$$

$$\frac{M_c}{M} = \frac{\gamma_a}{(\gamma_a-1)4}$$

Pokaži, da obstaja kritična temperatura, nad katero bo delno ioniziran vodik vedno dinamično stabilen in izračunaj to temperaturo.

$T = 2,75 \cdot 10^4$ K

$$\gamma_a(x,T) = \frac{5 + A^2 x(1-x)}{3 + Bx(1-x)}$$

$$A = \frac{5}{2} + \frac{\chi}{kT}$$

$$B = \frac{3}{2} + \left(\frac{\chi}{kT} + \frac{3}{2}\right)^2$$

$$x = \frac{n_+}{n_0 + n_+}$$ stopnja ionizacije

χ = ionizacijski potencial

$x_{min} = 0,5$ (minimum) $\longrightarrow \dfrac{d^2\gamma_a}{dx^2} > 0$

$$\gamma_{a,min}(T) = \frac{5 + A^2 \cdot \frac{1}{4}}{3 + B \cdot \frac{1}{4}}$$

$$\gamma_{a,min} = \frac{4}{3}$$

$$\frac{4}{3} = \frac{5 + \left(\frac{5}{2} + \frac{\chi}{kT}\right)^2 \cdot \frac{1}{4}}{3 + \left(\frac{3}{2} + \left(\frac{3}{2} + \frac{\chi}{kT}\right)^2\right) \cdot \frac{1}{4}}$$

$$u = \frac{\chi}{kT}$$

$$4u^2 - 12u - 63 = 0$$

$u \longrightarrow \underline{T = 2,75 \cdot 10^4 \text{ K}}$