Privzemi, da ima zvezda maso M in radij R ter da ima njeno jedro maso M_1 in radij R_1. Porazdelitev gostote naj opisujeta sledeči enačbi:

$$
\begin{aligned}
\rho(r) &= \rho_c - (\rho_c - \rho_1)\left(\frac{r}{R_1}\right)^2 za 0 \leq r \leq R_1 \\
\rho(r) &= \rho_1 \frac{(R_1/r)^3 - (R_1/R)^3}{1 - \left(\frac{R_1}{R}\right)^3} za R_1 \leq r \leq R
\end{aligned} \tag{1}
$$

kjer je središčna gostota ρ_c in $\rho_1 = \rho(R_1)$. Izračunaj odvisnost razmerja R/R_1 od $x_1 = \rho_c/\rho_1$ in $y_1 = M/M_1$. Izračunaj razmerje za $x_1 = 10$ in $y_1 = 7,5$, v skladu s Schönberg-Chandrasekharjevo relacijo $M_c/M \leq 0,37\left(\frac{\mu_{\mathrm{env}}}{\mu_c}\right)^2$.

Masa jedra

$$M_1 = \int_0^{R_1} 4\pi r^2 \rho(r)\, dr = 4\pi \int_0^{R_1} r^2\left(\rho_c - (\rho_c - \rho_1)\left(\frac{r}{R_1}\right)^2\right) dr =$$

$$= 4\pi \frac{R_1^3}{3}\rho_c - 4\pi(\rho_c - \rho_1)\frac{R_1^{\cancel{5}\,3}}{5} \cdot \frac{1}{\cancel{R_1^2}}$$

$$= 4\pi R_1^3\left(\frac{\rho_c}{3} - \frac{\rho_c}{5} + \frac{\rho_1}{5}\right) =$$

$$= \frac{4\pi R_1^3}{5}\left(\frac{2}{3}\rho_c + \rho_1\right)$$

Masa ovojnice

$$M - M_1 = \int_{R_1}^{R} 4\pi r^2 \rho_1 \frac{(R_1/r)^3 - (R_1/R)^3}{1 - (R_1/R)^3}\, dr \qquad u = 1 - \left(\frac{R_1}{R}\right)^3$$

$$= \int_{R_1}^{R} 4\pi r^2 \rho_1 \left[\left(\frac{R_1}{r}\right)^3 \cdot \frac{1}{u} - \left(\frac{R_1}{R}\right)^3 \frac{1}{u}\right] dr$$

$$= \frac{4\pi\rho_1}{1 - \left(\frac{R_1}{R}\right)^3} \int_{R_1}^{R} \left[\frac{R_1^3}{r} - r^2\left(\frac{R_1}{R}\right)^3\right] dr$$

$$= \frac{4\pi\rho_1}{1 - (R_1/R)^3}\left[R_1^3 \ln\frac{R}{R_1} - \frac{R^3}{3}\left(\frac{R_1}{R}\right)^3 + \frac{R_1^3}{3}\left(\frac{R_1}{R}\right)^3\right]$$

$$= \frac{4\pi\rho_1}{1-(R_1/R)^3}\left[R_1^3 \ln\frac{R}{R_1} - \frac{R^3}{3}\left(\frac{R_1}{R}\right)^3 + \frac{R_1^3}{3}\left(\frac{R_1}{R}\right)^3\right]$$

$$= 4\pi\rho_1 R_1^3 \left[\frac{\ln R/R_1}{1-\left(\frac{R_1}{R}\right)^3} - \frac{(R^3-R_1^3)}{3R^3\left(1-\left(\frac{R_1}{R}\right)^3\right)}\right]$$

$$M - M_1 = 4\pi\rho_1 R_1^3 \left[\frac{\ln R/R_1}{1-\left(\frac{R_1}{R}\right)^3} - \frac{\cancel{1-\left(\frac{R_1}{R}\right)^3}}{3\cancel{\left(1-\left(\frac{R_1}{R}\right)^3\right)}}\right]$$

$$\underline{\frac{R}{R_1}(x_1, y_1) = ?} \quad x_1 = \frac{\rho_c}{\rho_1} \; ; \; y_1 = \frac{M}{M_1}$$

$$\frac{M-M_1}{M_1} = \frac{M}{M_1} - 1 = y_1 - 1 = \frac{\cancel{4\pi}\rho_1 \cancel{R_1^3}\left[\frac{\ln(R/R_1)}{1-(R_1/R)^3} - \frac{1}{3}\right]}{\frac{\cancel{4\pi R_1^3}}{5}\left[\frac{2}{3}\rho_c + \rho_1\right]}$$

$$y_1 - 1 = \frac{\cancel{\rho_1}\left[\frac{\ln(R/R_1)}{1-(R_1/R)^3} - \frac{1}{3}\right]}{\frac{\cancel{\rho_1}}{5}\left[\frac{2}{3}x_1 + 1\right]}$$

$$\frac{\ln(R/R_1)}{1-\left(\frac{R_1}{R}\right)^3} = (y_1 - 1)\left(\frac{2}{3}x_1 + 1\right)\cdot\frac{1}{5} + \frac{1}{3}$$

če $R_1 \ll R$:

$$1 - \left(\frac{R_1}{R}\right)^3 \simeq 1$$

$$\frac{R}{R_1} \approx e^{\left[(y_1-1)\left(\frac{2}{3}x_1+1\right)\frac{1}{5} + \frac{1}{3}\right]}$$

$$x_1 = 10 \quad y_1 = 7{,}5 \quad \longrightarrow \quad \frac{R}{R_1} \approx 3\cdot 10^4$$

(dodatna naloga) Predpostavi, da je jedro zvezde v kolapsu, ohranja pa uniformno gostoto (homologno krčenje). Pokaži, da je rešitev enačbe gibanja $|v| \propto r$.