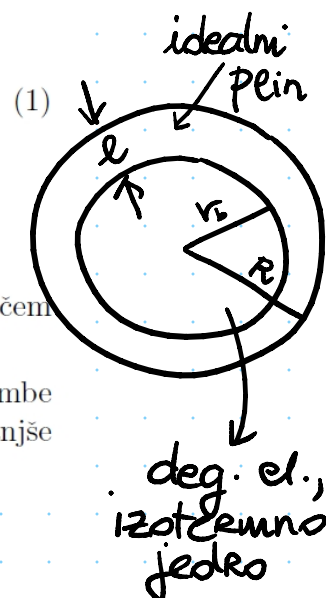


Profil temperature v zunanem delu plasti bele pritlikavke z maso M in radijem R opisuje funkcija

$$T(r) = \frac{4}{17} \frac{\mu m_H}{k} GM \left(\frac{1}{r} - \frac{1}{R} \right).$$

Pokaži sledeče:

- izpelji zgornjo enačbo;
- da je debelina te plasti $l = R - r_b \ll R$ (kjer je r_b mejni radij med središčem z degeneriranim plinom in zunanjo plastjo);
- izračunaj relativno spremembo debeline lupine, l_1/l_2 , če pride do spremembe (upada) izseva od $L_1 = 10^{-2} L_\odot$ in $L_2 = 10^{-4} L_\odot$ (če ne upoštevamo manjše spremembe v R).



a) $\frac{dp}{dr} = \frac{dp}{dT} \frac{dT}{dr} = - \rho \frac{GM}{r^2}$ (hidr. raon.)

$$\downarrow \quad p = \frac{\rho}{\mu m_H} kT \rightarrow \rho = \frac{p \mu m_H}{kT}$$

$$\frac{dp}{dT} \frac{dT}{dr} = - \frac{p \mu m_H}{kT} \frac{GM}{r^2} \quad | : \left(\frac{p}{T} \right)$$

$$\frac{dp}{p} \frac{T}{dT} \frac{dT}{dr} = - \frac{\mu m_H}{k} \frac{GM}{r^2}$$

$$\frac{d \ln p}{d \ln T} \cdot \frac{dT}{dr} = - \frac{\mu m_H}{k} \frac{GM}{r^2}$$

$$p(T) = \left(\frac{64 \pi a c k G}{51 \kappa_0 \mu m_H} \right)^{1/2} \left(\frac{M}{L} \right)^{1/2} T^{17/4}$$

$$\ln p = \ln A + \frac{17}{4} \ln T$$

$$\frac{d \ln p}{d \ln T} = \frac{17}{4}$$

$$\frac{17}{4} \frac{dT}{dr} = - \frac{\mu m_H}{k} \frac{GM}{r^2}$$

$$\frac{dT}{dr} = - \frac{4}{17} \frac{\mu m_H}{k} \frac{GM}{r^2}$$

$$b) \quad l = R - r_b \ll R$$

$$\frac{dT}{dr} = - \frac{4}{17} \frac{\mu_{MH}}{k} \frac{GM}{r^2}$$

$$\int_{T(r_b)}^{T(R)} dT = - \frac{4}{17} \frac{\mu_{MH}}{k} GM \int_{r_b}^R \frac{1}{r^2} dr$$

$$T(R) = 0$$

$$T(r_b) = T_c \quad (\text{izotermno jedro})$$

$$0 - T_c = - \frac{4}{17} \frac{\mu_{MH}}{k} GM \left(- \frac{1}{r} \right) \Big|_{r_b}^R$$

$$T_c = \frac{4}{17} \frac{\mu_{MH}}{k} GM \left(\frac{R - r_b}{r_b R} \right)$$

$$\frac{kT_c}{\mu_{MH}} = \frac{4}{17} \frac{GM}{R} \left(\frac{R - r_b}{r_b} \right)$$

$$\left(= \frac{p_i}{p} \right) \quad \left(= \frac{p}{p} \right)$$

$$p \approx p_e \gg p_i$$

$$\frac{p_i}{p_e} \ll 1 \quad \frac{R - r_b}{r_b} \ll 1 \quad \Rightarrow \quad \frac{R - r_b \ll r_b < R}{R - r_b \ll R}$$

$$\underline{l \ll R}$$

$$c) \quad l = R - r_b \ll R$$

$$r_b \approx R$$

$$\frac{L}{M} = \frac{64\pi a c G K_1^{13} \mu_{MH}^4}{51 k_p^4 \kappa_0 \mu_e^2} T_c^{7/2}$$

$$\frac{l_1}{l_2} = ?$$

$$L_1 = 10^{-2} L_0$$

$$L_2 = 10^{-4} L_0$$

$$\frac{k T_c}{\mu_{MH}} = \frac{4}{17} \frac{GM}{R} \left(\frac{R - r_b}{r_b} \right) \quad (\text{iz točke b})$$

$$T_c = \frac{4}{17} \frac{\mu_{MH}}{k} \frac{GM}{R} \cdot \frac{l}{R}$$

$$T_c \propto l$$

$$L \propto T_c^{7/2}$$

$$L \propto l^{7/2} \rightarrow l \propto L^{2/7}$$

$$\frac{l_1}{l_2} = \left(\frac{L_1}{L_2} \right)^{2/7} = 100^{2/7} = 3,73$$