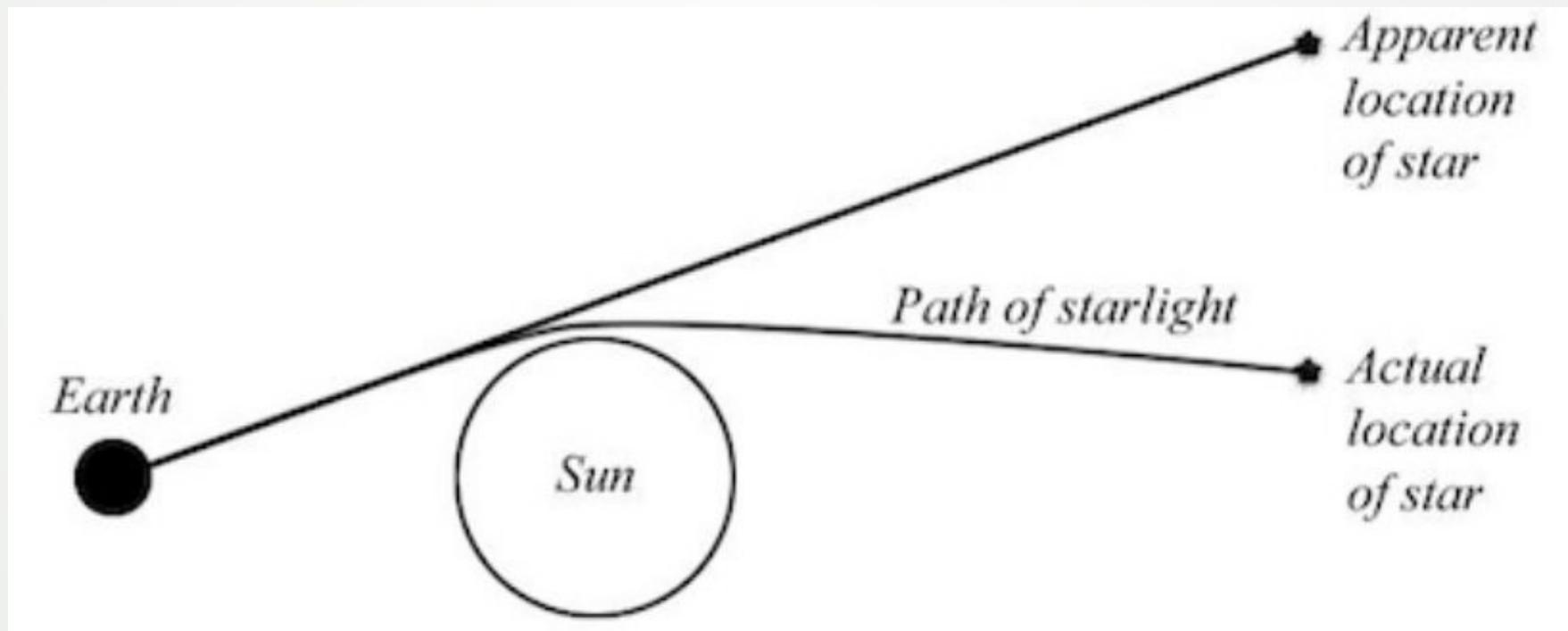


# Gravitacijsko lečenje

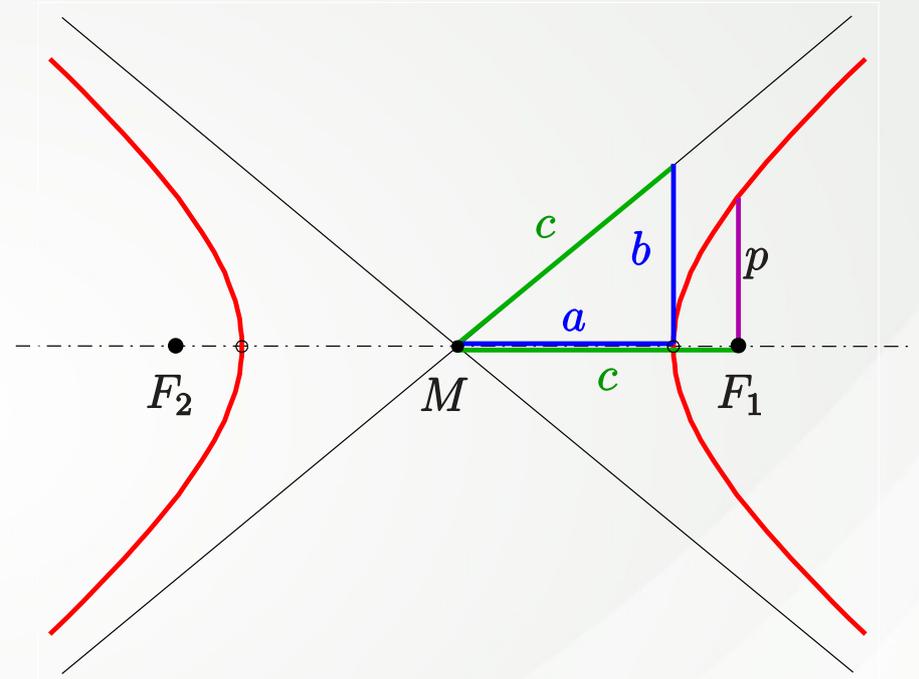
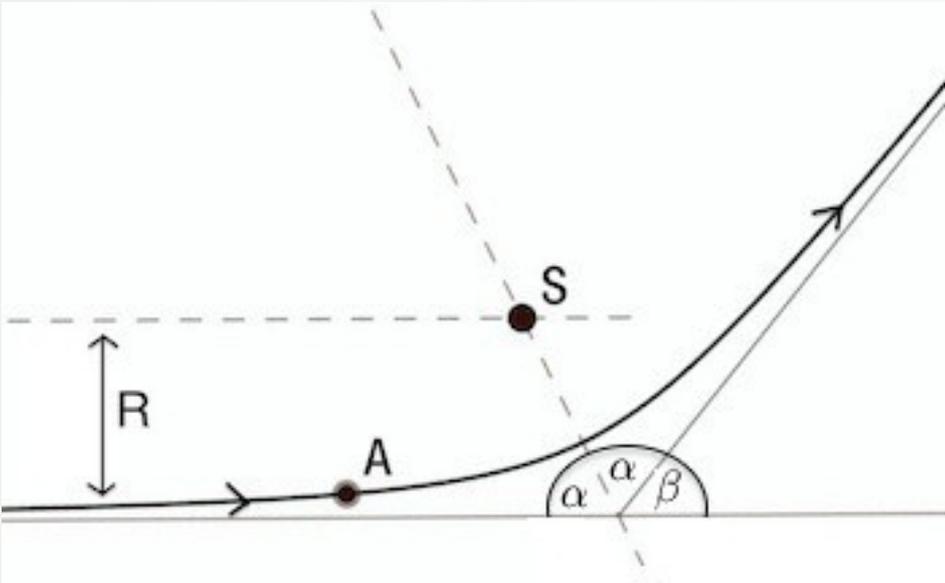
(intermezzo)

**Viri:** Jones, Lambourne (Chap. 1), Maoz (Chap. 6)

# Newtonova in Einsteinova teorija na preizkusu



# Odklonski kot (Newton)



asteroid (A) se giblje mimo Sonca (S);  
kot  $\beta$  imenujemo *odklonski kot*,  
razdaljo  $R$  pa imenujemo  
*vpadni parameter*

$$\beta = \frac{2GM}{c^2 R}$$

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

enačba hiperbole  
 $a$  – velika polos  
 $b$  – mala polos

# Odklonski kot (Einstein)

Einsteinova enačba polja

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

tenzor, povezan z metriko  
(ukrivljenost prostorčasa)

tenzor, ki opisuje  
porazdelitev energije  
in gibalne količine

Opis prostor-časa z metriko:

**Minkowski:**

$$(ds)^2 = (cdt)^2 - (dx)^2 - (dy)^2 - (dz)^2$$

$$(ds)^2 = (cdt)^2 - (dr)^2 - (rd\theta)^2 - (r \sin \theta d\phi)^2$$

sferni koord.  
sistem

**Schwarzschild:**

$$(ds)^2 = \left(1 - \frac{2GM}{rc^2}\right) (cdt)^2 - \left(1 - \frac{2GM}{rc^2}\right)^{-1} (dr)^2 - (rd\theta)^2 - (r \sin \theta d\phi)^2$$

*geometrija prostor-časa okrog statične, sferne masne distribucije*

$$\beta = \frac{4GM}{c^2 R}$$

# Potrditvev splošne relativnosti

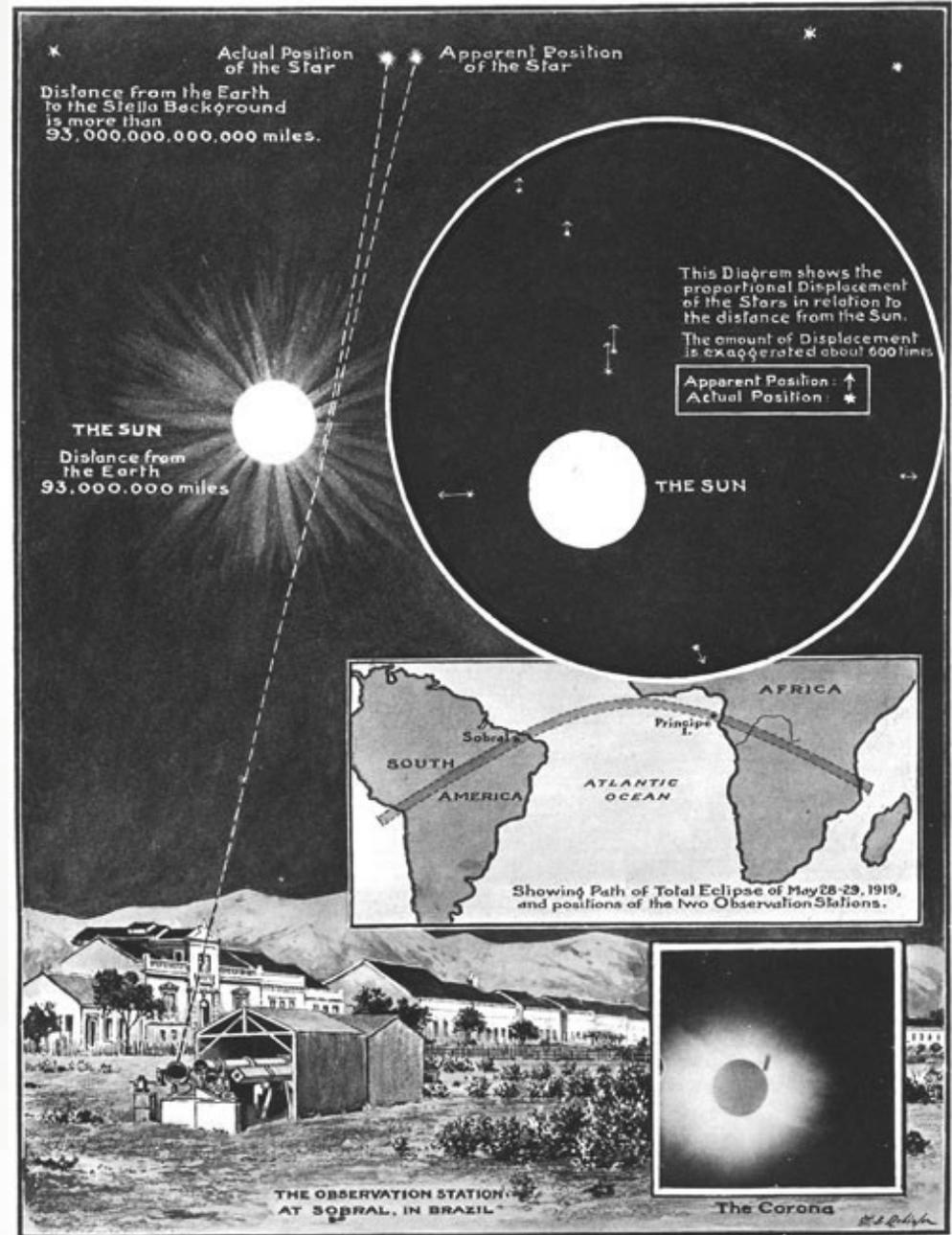
- 1919 - Eddington potrdi Einsteinovo teorijo
- opazovanje Sončevega mrka iz otoka Principe in mesta Sobral

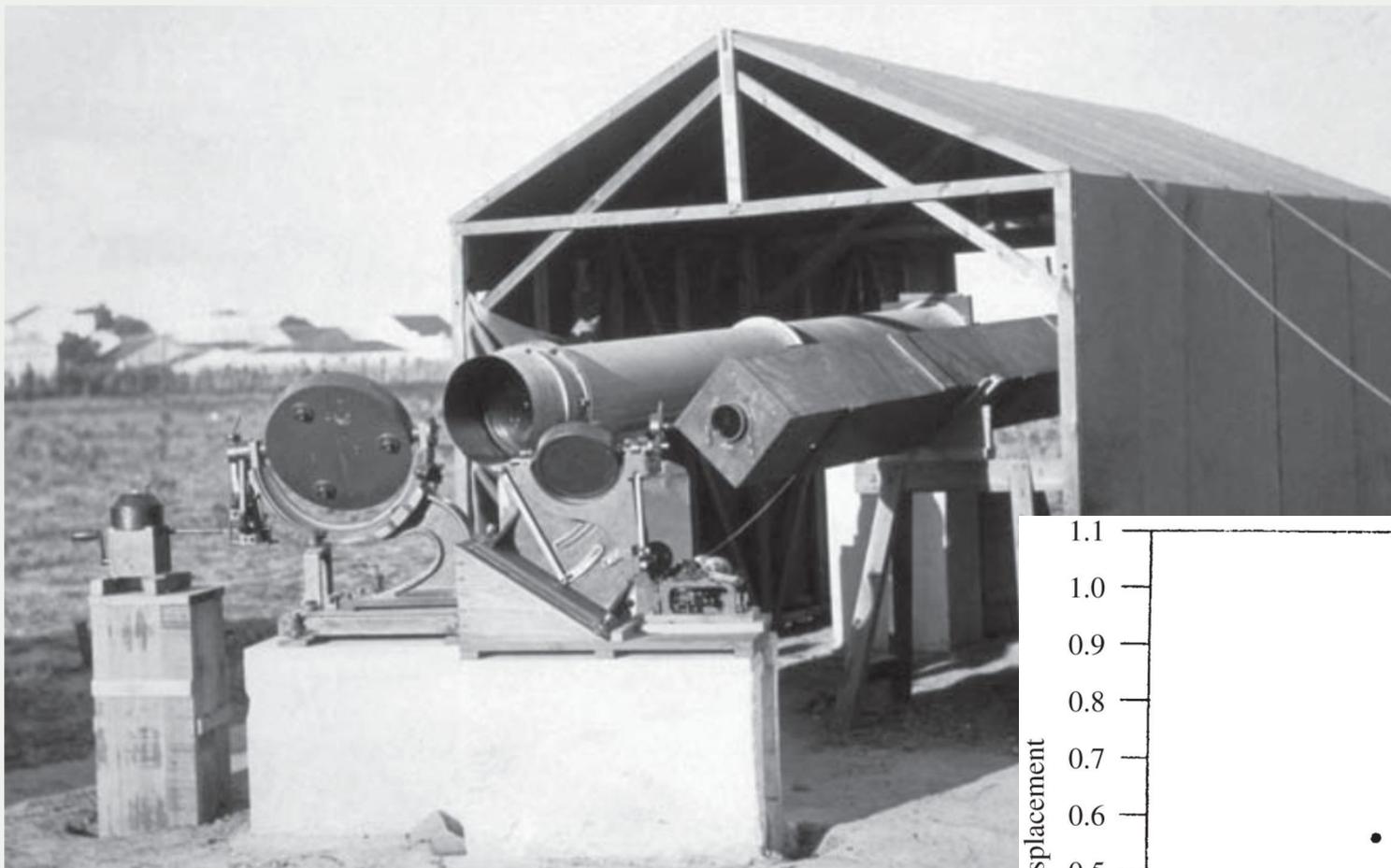
LIGHTS ALL ASKEW  
IN THE HEAVENS

Men of Science More or Less  
Agog Over Results of Eclipse  
Observations.

EINSTEIN THEORY TRIUMPHS

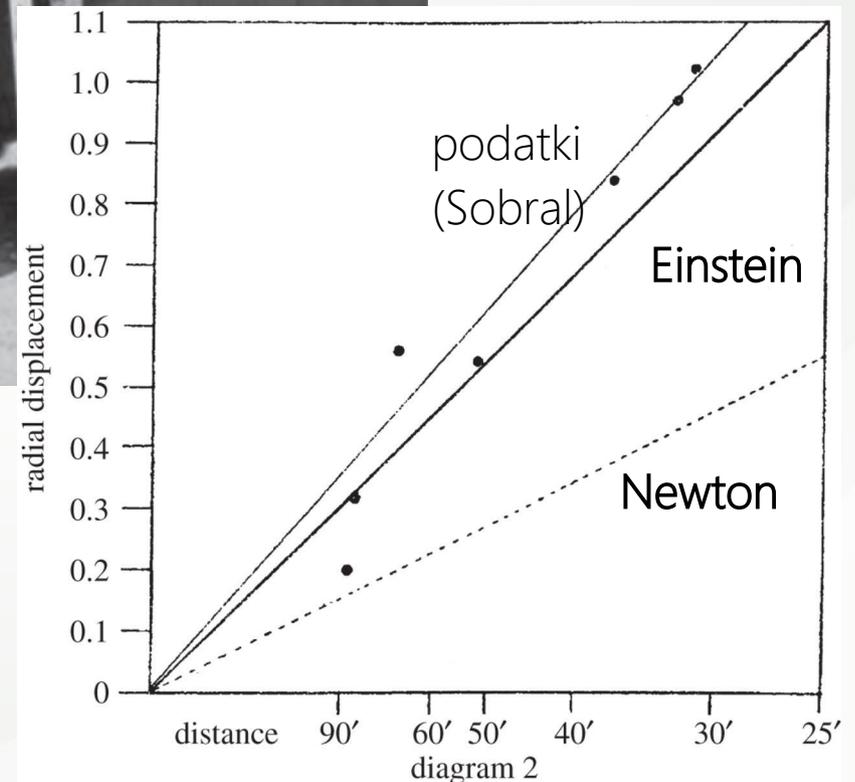
Stars Not Where They Seemed  
or Were Calculated to be,  
but Nobody Need Worry.





Postavitev teleskopov za opazovanje Sončevega mrka v mestu Sobral (Brazilija): 16-inčna leča in celostat (levo) ter 4-inčni teleskop v kvadratni cevi (desno)

Principe (Zahodna Afrika):  $1,61 \pm 0,31''$   
 Sobral (Brazilija):  $1,98 \pm 0,12''$  (4-inčni teleskop)



rezultati za podatke iz Sobrala  
 (abscisna os prikazuje oddaljenost od središča Sonca)

# Predlagano branje za ozadje meritev Eddingtona in njegovih sodelavcev:

Malcolm Longair, *Bending space-time: a commentary on Dyson, Eddington and Davidson (1920) 'A determination of the deflection of light by the Sun's gravitational field'*

<https://royalsocietypublishing.org/doi/full/10.1098/rsta.2014.0287>

Review



**Cite this article:** Longair M. 2015 Bending space–time: a commentary on Dyson, Eddington and Davidson (1920) ‘A determination of the deflection of light by the Sun’s gravitational field’. *Phil. Trans. R. Soc. A* **373**: 20140287.  
<http://dx.doi.org/10.1098/rsta.2014.0287>

One contribution of 17 to a theme issue ‘Celebrating 350 years of *Philosophical Transactions*: physical sciences papers’.

**Subject Areas:**

relativity, astrophysics

**Keywords:**

general relativity, light bending by the Sun, eclipse expedition 1919, gravitational lensing, Eddington

**Author for correspondence:**

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The featured article can be viewed at  
<http://dx.doi.org/10.1098/rsta.1920.0009>.

## Bending space–time: a commentary on Dyson, Eddington and Davidson (1920) ‘A determination of the deflection of light by the Sun’s gravitational field’

Malcolm Longair

Cavendish Laboratory, JJ Thomson Avenue, Cambridge CB3 0HE, UK

The famous eclipse expedition of 1919 to Sobral, Brazil, and the island of Principe, in the Gulf of Guinea, led by Dyson, Eddington and Davidson was a turning point in the history of relativity, not only because of its importance as a test of Einstein’s General Theory of Relativity, but also because of the intense public interest which was aroused by the success of the expedition. The dramatic sequence of events which occurred is reviewed, as well as the long-term impact of its success. The gravitational bending of electromagnetic waves by massive bodies is a subject of the greatest importance for contemporary and future astronomy, astrophysics and cosmology. Examples of the potential impact of this key tool of modern observational astronomy are presented. This commentary was written to celebrate the 350th anniversary of the journal *Philosophical Transactions of the Royal Society*.

### 1. Einstein and bent space–time

The famous expedition to measure the deflection of the positions of stars caused by the curvature of space–time in the gravitational field of the Sun had a profound

# Odklonski kot

- Odklonski kot za žarek, ki gre tik mimo roba Sonca:

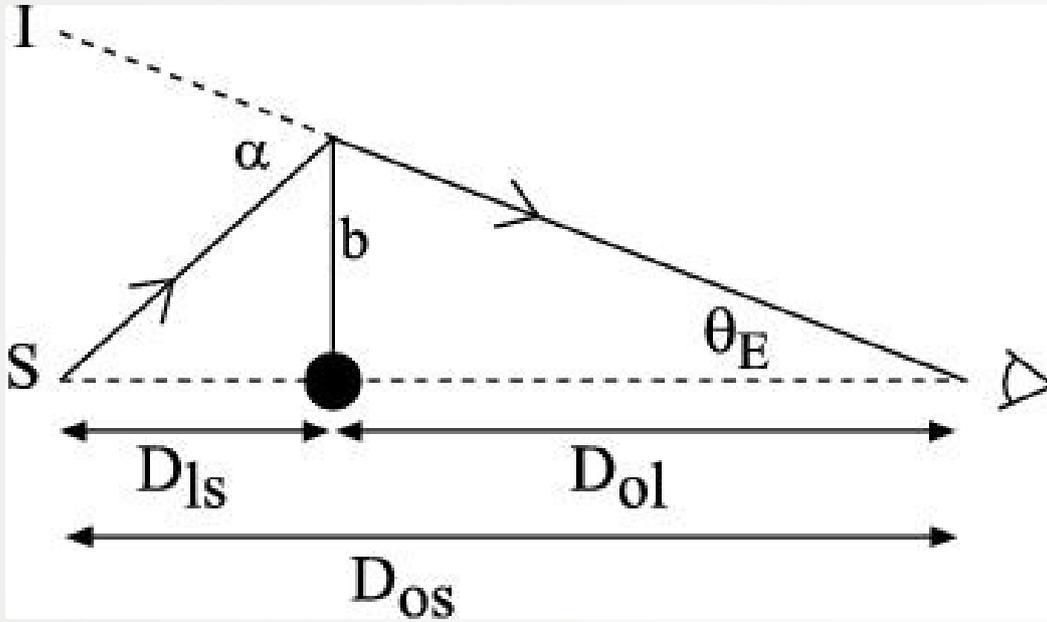
NEWTON:

$$\beta = 0.87''$$

EINSTEIN:

$$\beta = 1.75''$$

# Osnove gravitacijskega lečenja



$\theta_E$  = Einsteinov obroč

$$SI = \alpha D_{ls} = \theta_E D_{os}$$

$$b = D_{ol} \theta_E$$

$$\alpha = \frac{4GM}{c^2 b}$$

$$\frac{4GM}{c^2} \frac{1}{D_{ol} \theta_E} D_{ls} = \theta_E D_{os}$$

S = oddaljeni izvor (angl. *source*)

D<sub>ls</sub> = oddaljenost LEČA - IZVOR

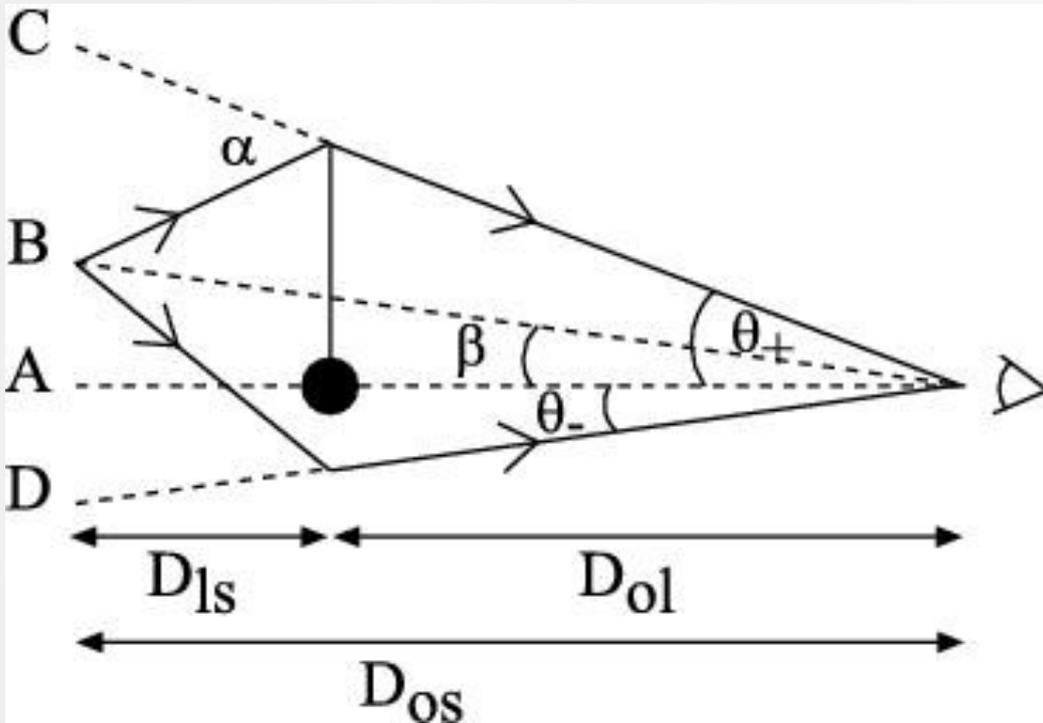
D<sub>ol</sub> = oddaljenost OPAZOVALEC - LEČA

D<sub>os</sub> = oddaljenost OPAZOVALEC - IZVOR

$$\theta_E = \left( \frac{4GM}{c^2} \frac{D_{ls}}{D_{ol} D_{os}} \right)^{1/2}$$

*enačba leče za točkasto maso*

# Osnove gravitacijskega lečenja



$D_{ls}$  = oddaljenost LEČA - IZVOR

$D_{ol}$  = oddaljenost OPAZOVALEC - LEČA

$D_{os}$  = oddaljenost OPAZOVALEC - IZVOR

## SPLOŠEN PRIMER

$\beta$  = kot med izvorom (B) in lečo

$\theta$  = kot med lečo in sliko (C ali D)

$$AB + BC = AC$$

$$\beta D_{os} + \alpha D_{ls} = \theta D_{os}$$

$$\beta = \theta - \frac{4GM}{c^2} \frac{1}{D_{ol}\theta} \frac{D_{ls}}{D_{os}}$$

$$\theta^2 - \beta\theta - \theta_E^2 = 0$$

*kvadratna enačba za točkasto maso (izvor izven osi)*

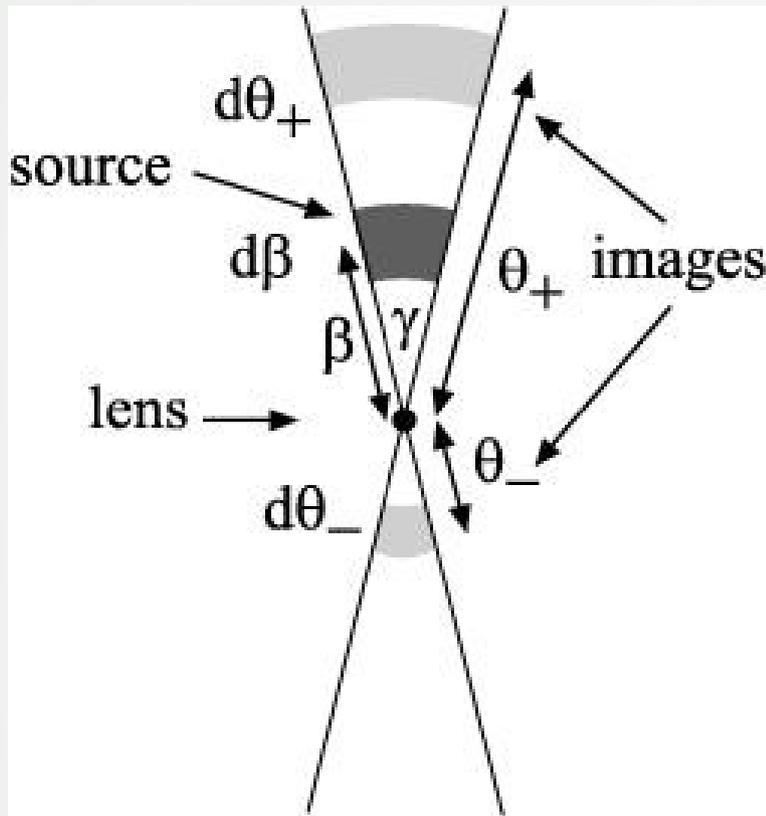
# Gravitacijsko lečenje s kozarcem vina

GRAVITATIONAL LENSING  
WITH  
A WINE GLASS

<https://www.youtube.com/watch?v=vLp6CwElGP4>

Perimeter Classroom Activity: Gravitational Lensing with a Wine Glass

# Osnove gravitacijskega lečenja



$d\beta$  in  $d\theta$  prikazujeta radialno  
RAZTEZANJE slike

KJE SE BOSTA POJAVILI SLIKI?

- *povečava* (magnification)

$$a_p = \frac{\theta_p d\theta_p}{\beta d\beta} \quad a_m = \frac{\theta_m d\theta_m}{\beta d\beta}$$

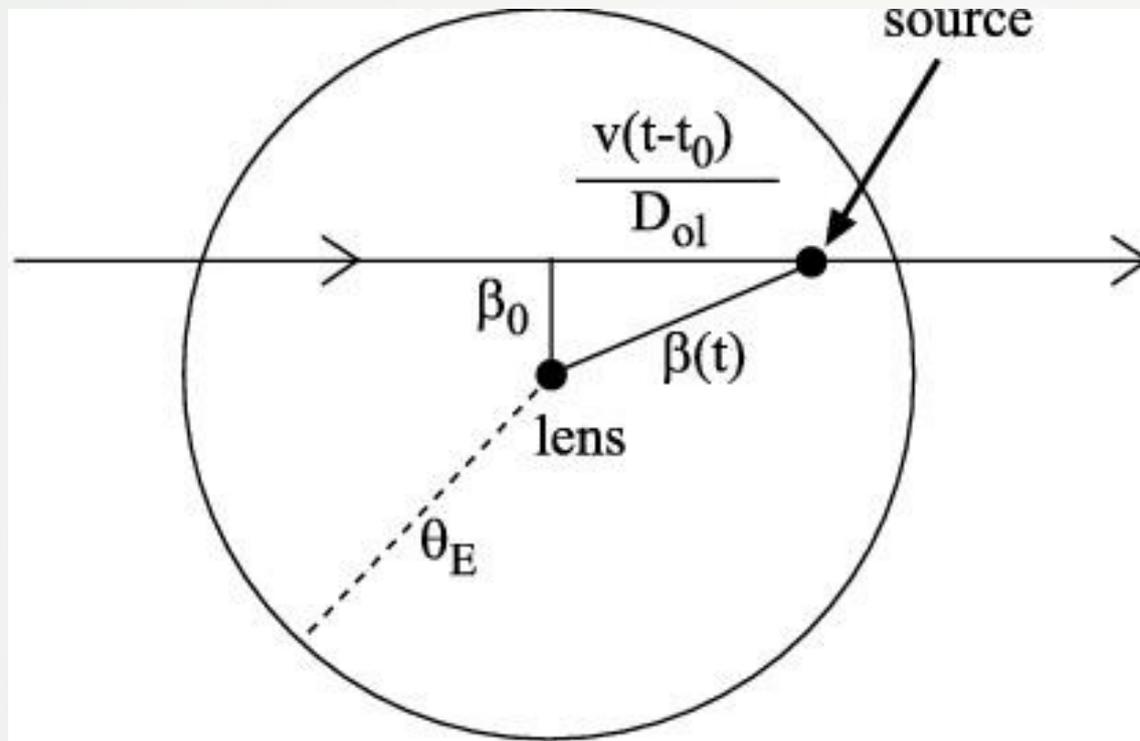
- *ojačenje* (amplification)

*povečana slika bo zgedala  
svetlejša, ker se ohrani  
površinska svetlost*

$$\theta^2 - \beta\theta - \theta_E^2 = 0$$

*kvadratna enačba za  
točkasto maso (izvor izven osi)*

# Mikrolečenje



$$\beta(t) = \left[ \beta_0^2 + \frac{v^2}{D_{ol}^2} (t - t_0)^2 \right]^{1/2}$$

# Osnove gravitacijskega lečenja

video: ***Single Lens Microlensing***

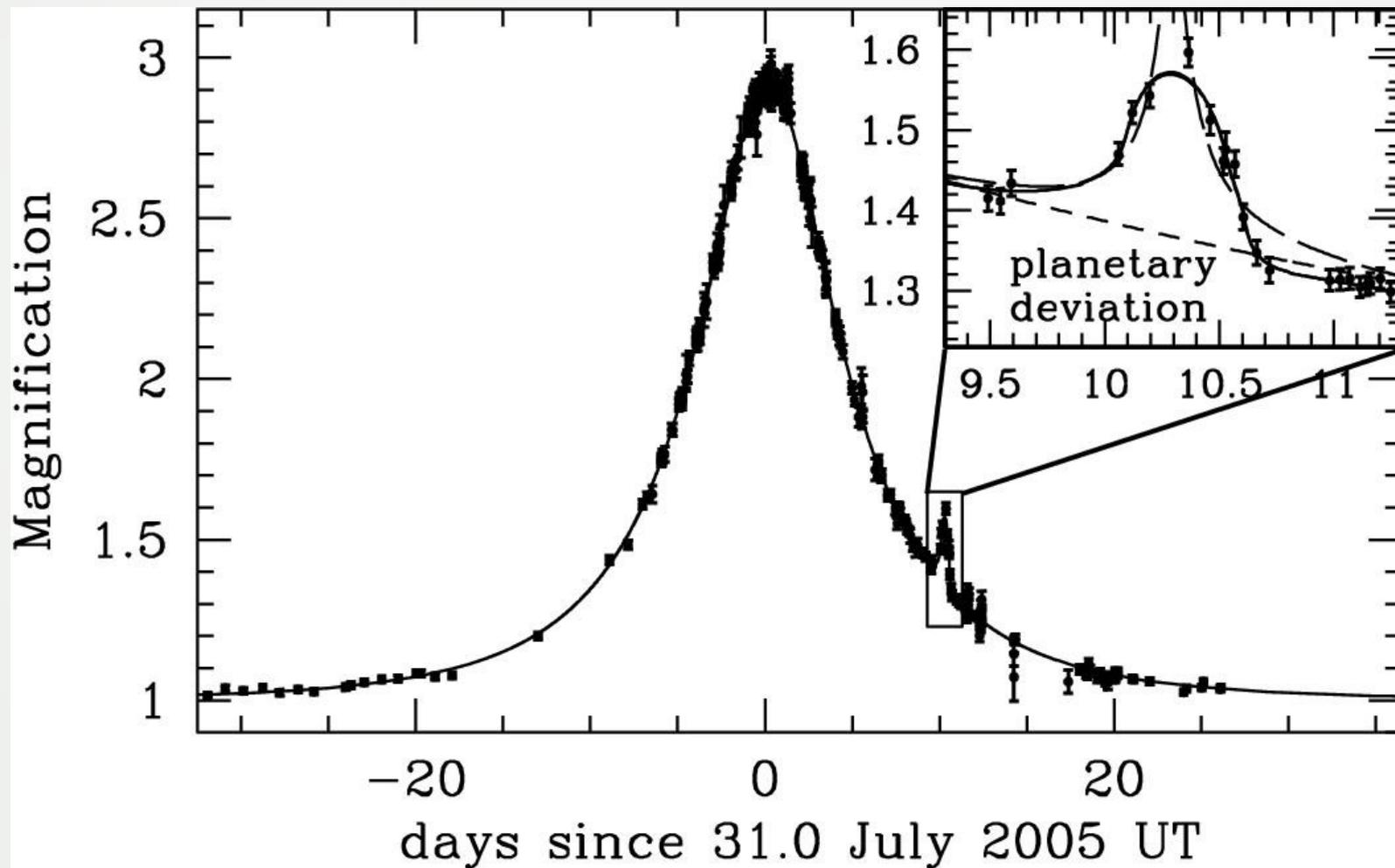
<http://www.astronomy.ohio-state.edu/~gaudi/movies.html>

# Eksperiment mikrolečenja v Velikem Magelanovem oblaku



Prvi predlog: Paczynski, 1986

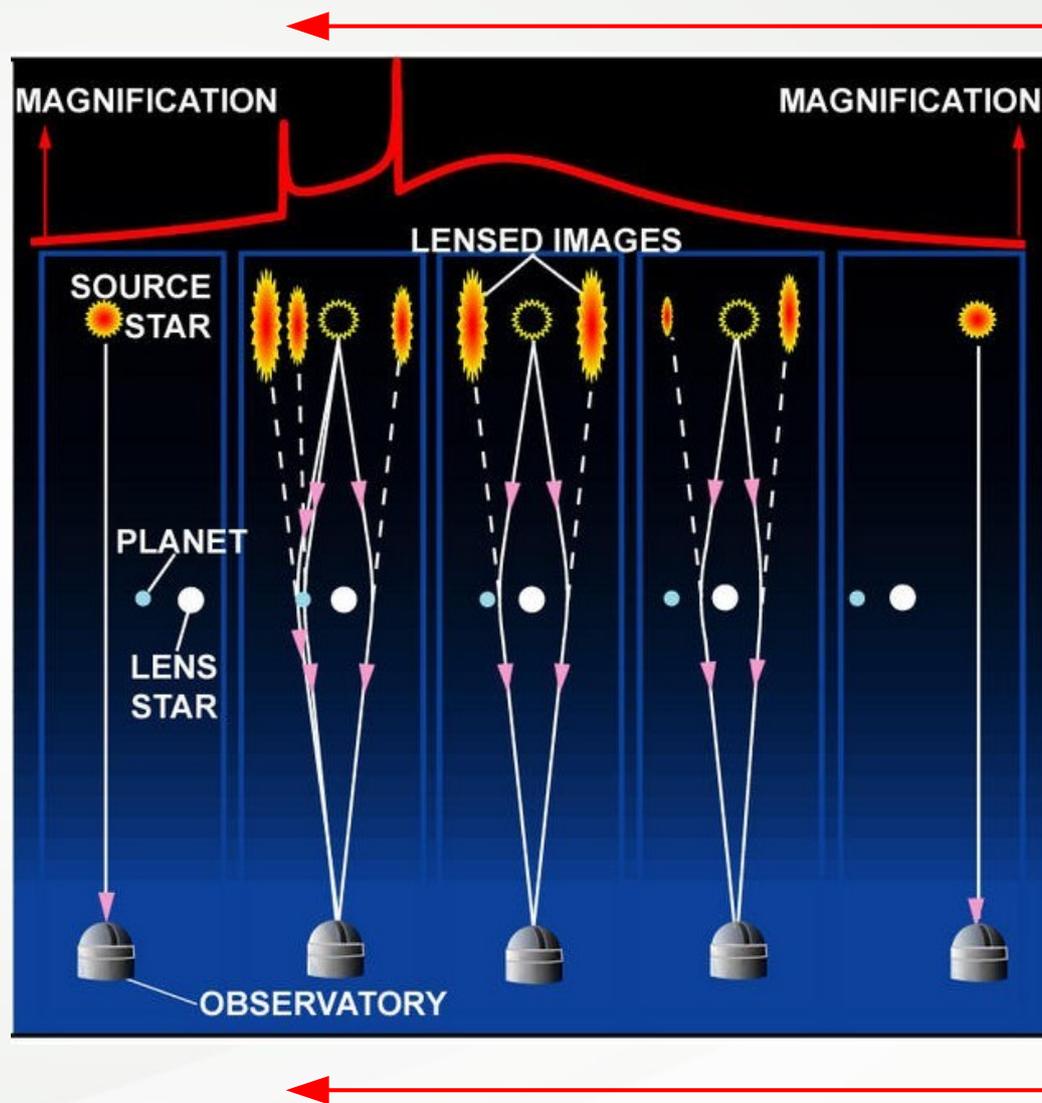
# Osnove gravitacijskega lečenja



# Osnove gravitacijskega lečenja

video: *Planetary Microlensing*

<http://www.astronomy.ohio-state.edu/~gaudi/movies.html>



<http://www.astronomy.ohio-state.edu/~gaudi/movies.html>