

### Differentiability

1. Discuss the differentiability of each of the following.

- (a) A constant function.
- (b) The monomial function  $f : \mathbb{R} \rightarrow \mathbb{R}$ ,  $f(x) = x^n$ .
- (c) The absolute value function.
- (d)  $f(x) = \operatorname{sgn}(x)x^2$
- (e)  $f(x) = |x^3|$

2. Discuss the differentiability of the function  $f : \mathbb{R} \rightarrow \mathbb{R}$ ,

$$f(x) = \begin{cases} x^2 \cos \frac{1}{x} & \text{if } x \neq 0, \\ 0 & \text{if } x = 0 \end{cases}.$$

3. Rules for differentiation: derivation of quotient rule.

- a) Write down the product rule.
- b) Write down the chain rule.
- c) Is the function  $f(x) = \frac{1}{x}$  differentiable? Are there any points at which it is not? What is its derivative?
- d) Now let  $g$  and  $h$  be differentiable functions and define  $f(x) = \frac{g(x)}{h(x)}$  for all  $x$ . At what points is  $f$  differentiable, and what is its derivative at those points?

4. Find the derivatives of the following functions.

- a)  $\frac{2x^3 - 5x + 3}{9x + 4}$
- b)  $e^{x \sin x}$
- c)  $\frac{x + \cos x}{x^2 + 3x}$

5. Are the following functions differentiable at the given points? If so, evaluate the derivative at this point.
- a)  $\tan x$ , at  $x = \frac{3}{4}\pi$ .
  - b)  $\tan^{-1} x$ , at  $x = 2\pi$ .
  - c)  $\frac{2x^3 - 5x + 3}{9x + 4}$ , at  $x = -0.44444$
- 6.
- a) Let  $f$  and  $g$  be differentiable functions that are inverses of each other. By definition, this means that we have  $f(g(x)) = x$  at every  $x$ . In other words,  $x$  and  $f \circ g$  both represent the same function of  $x$ . Differentiate both sides of the latter equation to get two ways of writing the derivative of this function.
  - b) Now let  $y$  be a variable satisfying  $y = g(x)$  for all  $x$ . Let us substitute this into the above to change it from an equation in terms of functions of  $x$  into one in terms of functions of  $y$ .
  - c) Now if we have a function  $f(x)$  which we do not know how to differentiate, but of whose *inverse* we do know the derivative, we have a formula for the relationship between the derivatives of the function and its inverse. With this in mind, what is the derivative of the function  $f(x) = \ln x$ ?
7. Plot the graph of the function  $f(x) = 2x^5 - 10x^3$ .