

Integration

1. Use graphical evidence and known area facts to find:

a) $\int_1^3 (1 + 4x) dx$

b) $\int_{-1}^2 (2 - x) dx$

c) $\int_{-2}^2 \sqrt{4 - x^2} dx$

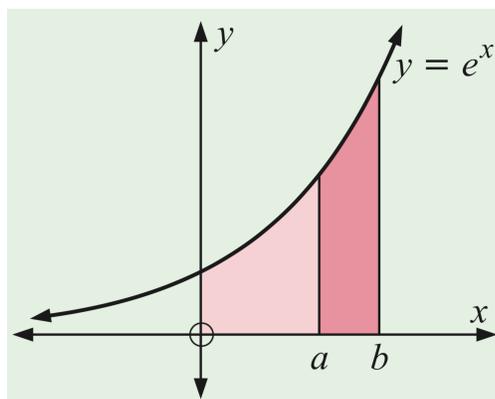
2. Find $\frac{d}{dx}(\ln x)^2$ and use it to compute $\int \frac{\ln x}{x} dx$.

3. How would we describe in English the difference between a *definite integral* and an *indefinite integral (antiderivative)*?

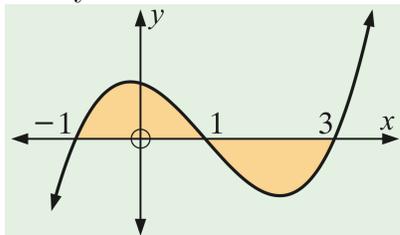
4. The area of the region defined by $y = x^2$ and $y = mx$ is $\frac{4}{3}$ units². Find m .

5. If $\int_1^3 f(x) dx = 2$ and $\int_1^6 f(x) dx = -3$, find $\int_3^6 f(x) dx$.

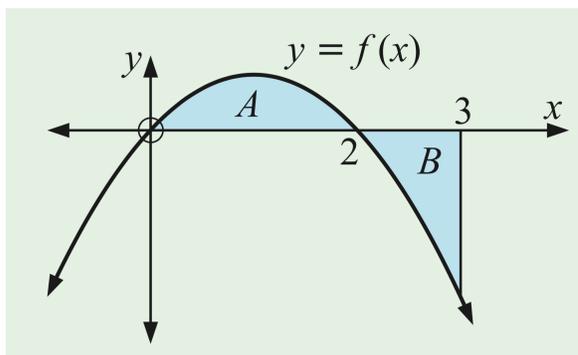
6. Find a given that the area of the region between $y = e^x$ and the x -axis from $x = 0$ to $x = a$ is 2 units². Then determine b , given that the area of the region between $x = a$ and $x = b$ is also 2 units².



7. Is it true that $\int_{-1}^3 f(x)dx$ represents the area of the shaded region? Explain your answer briefly.



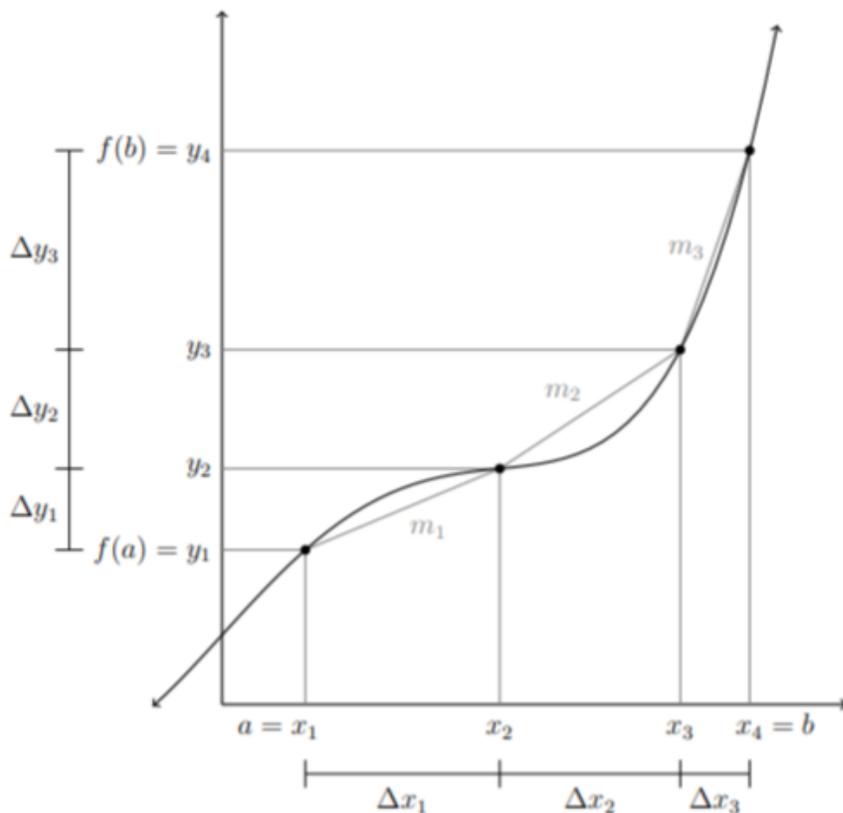
8. The graph of $y = f(x)$ is illustrated below. It is known that $\int_0^3 f(x)dx = 3$.



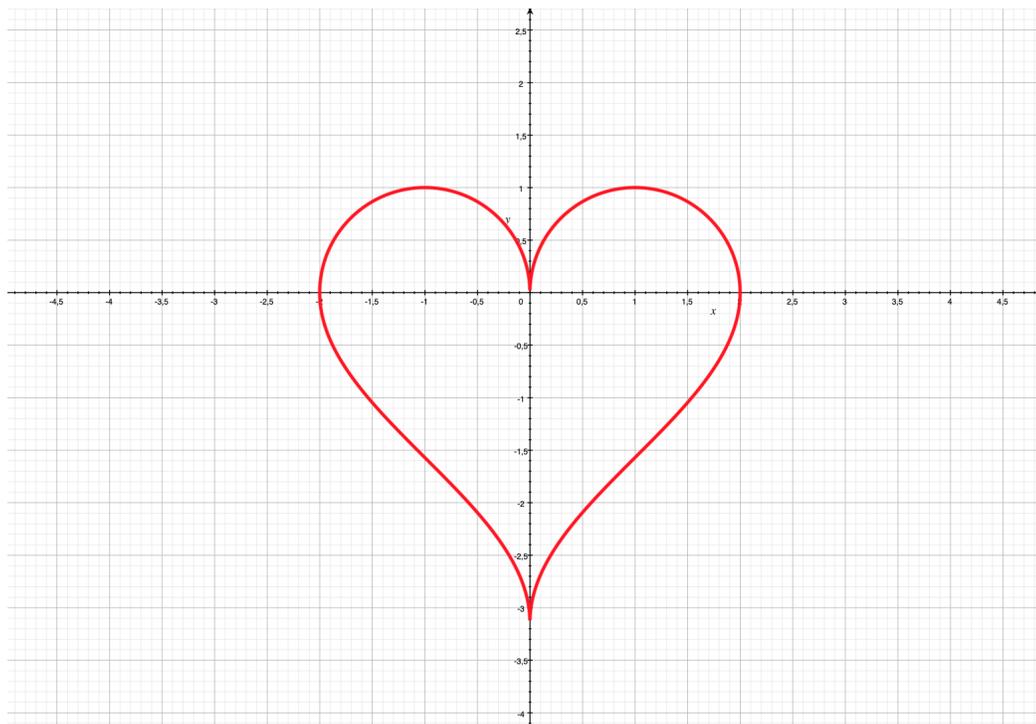
- a) What can be deduced about the areas A and B ?

- b) Can $\int_0^2 f(x)dx = 2$?

9. Consider the graph of $y = f(x)$ below. Assume f is differentiable on the interval $[a, b]$.
- If we define $\Delta y = \Delta y_1 + \Delta y_2 + \Delta y_3$, then how do we write Δy in terms of the m_i and the Δx_i ?
 - By a theorem called the *mean value theorem*, we know that for each i there exists a c_i with $x_i \leq c_i \leq x_{i+1}$ and $f'(c_i) = m_i$. How can we rewrite the above formula for Δy taking this into account?
 - What would happen to Δy if, instead of just three Δx_i between $x = a$ and $x = b$, there were more and each one was smaller? What happens when the size of each Δx_i tends to zero?



10. Compute the area of the heart shape below.



11. Evaluate the improper integral $\int_0^\infty x e^{-x} dx$.