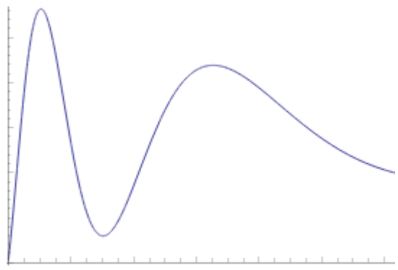
Expressing Mathematics in English - Second Midterm Test

10 May 2021

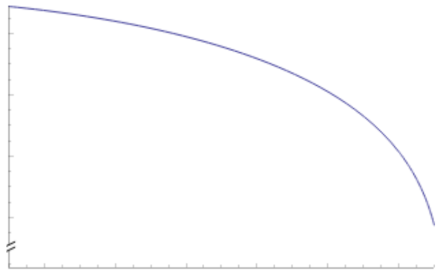
You have 90 minutes to answer the following six questions. Please write neatly, and please ensure that you follow all instructions on the "Information about the midterm test" *page on the course webpage. Before submitting, remember to check that all pages of your answer show your name and student ID, and are numbered.*

(1) For each picture, choose an appropriate description of the function from the list below.

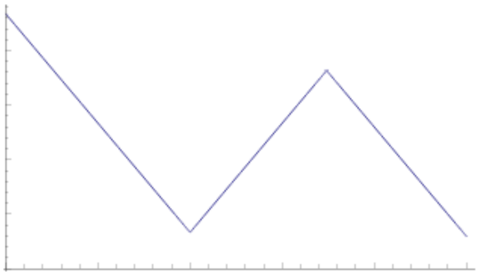
(a)



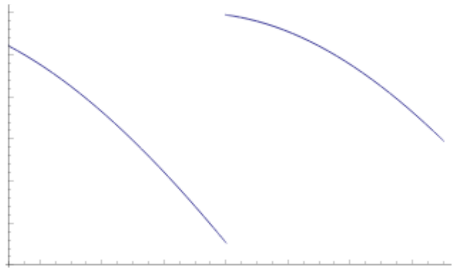
(b)



(c)



(d)



function with two stationary points; decreasing function;
convex function; discontinuous function; function with one point of inflection;
function with three stationary points; non-differentiable continuous function;

(2)(a) Explain in words what we mean by the *modulus* and *argument* of a complex number, and illustrate your definition with a sketch of a complex number in an Argand diagram.

(b) Write down the complex number $\frac{5}{3} + \frac{5}{\sqrt{3}}\,\mathrm{i}$ in polar co-ordinates.

(3) Logically negate the following sentences.

(a) The sequence $a_1, a_2, \ldots$ is an arithmetic progression and the sequence $b_1, b_2, \ldots$ is not.

(b) There are species of birds that are unable to fly.

(4) Write down the words that appropriately fill in the blanks in the following paragraphs. Some of the blanked words appear more than once, in which case they are given the same letter (this may help you work out what they are); you do not have to write them again.

- To determine the ___(a)___ or divergence of an infinite ___(b)___ $\sum_{i=1}^{\infty} a_i$, certain tests can be applied. If the infinite ___(b)___ $\sum_{i=1}^{\infty} a_i$ ___(c)___, then it must be true that a_n ___(d)___ to 0 as n ___(e)___. Furthermore, if all the terms in a ___(b)___ are ___(f)___, then it must be true that the partial sums $\sum_{i=1}^{n} a_i$ increase as n ___(e)___, either approaching a finite ___(g)___ (we say the sequence ___(c)___), or growing unboundedly (we say the sequence ___(h)___). This observation leads to what is called the comparison ___(i)___: if $0 \leq a_n \leq b_n$ for ___(j)___ n and if $\sum_{n=1}^{\infty} b_n$ is convergent, then $\sum_{n=1}^{\infty} a_n$ also ___(c)___.

- The derivative of a ___(k)___ of a real ___(l)___ measures the rate of change of its output value with respect to a change in its ___(m)___ value. The derivative of a ___(k)___ at a chosen ___(m)___ value, when it exists, is the gradient of the ___(n)___ line to the graph of the ___(k)___ at that point. The process of finding a derivative is called ___(o)___. The ___(p)___ process is called antidifferentiation. The ___(q)___ theorem of ___(r)___ relates antidifferentiation with ___(s)___.

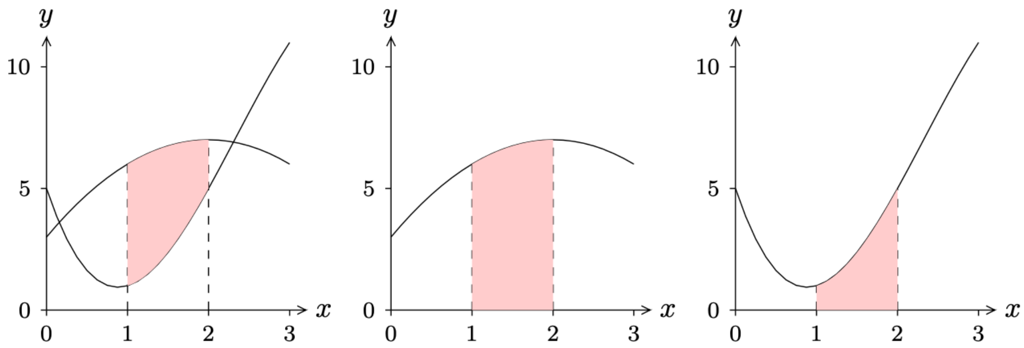
(5) Translate the Slovene texts below into English.

(a) *Standardni odklon je statistični kazalec, največkrat uporabljen za merjenje statistične razpršenosti enot. Z njim je moč izmeriti, kako razpršene so vrednosti, vsebovane v populaciji. Standardni odklon je definiran kot kvadratni koren variance, s čimer je v vsakem primeru dosežena pozitivna vrednost kazalca.*

(b) Geometrična sredina množice pozitivnih števil je n-ti koren zmnožka vseh elementov množice, kjer je n število elementov. Geometrična sredina množice je vedno manjša ali enaka aritmetični sredini množice. Obe sredini sta enaki, če so vsi elementi množice enaki.

(6) Read the text on the next page, and answer the multiple choice questions that follow.

The left-hand graph below shows the functions $f(x) = -x^2+4x+3$ and $g(x) = -x^3+7x^2-10x+5$, with the area below f and above g over the interval $1 \le x \le 2$ shaded. We calculate this area.



The middle and right-hand graphs show f and g respectively, with the areas under them shaded. It is clear that the area we want to calculate is the area under f minus the area under g:

$$\int_1^2 f(x)\,\mathrm{d}x \;-\; \int_1^2 g(x)\,\mathrm{d}x \ .$$

Since we have

$$\int_1^2 f(x)\,\mathrm{d}x \;-\; \int_1^2 g(x)\,\mathrm{d}x \;=\; \int_1^2 f(x) - g(x)\,\mathrm{d}x \ , \tag{1}$$

we can calculate the integral on the right:

$$\begin{aligned}
\int_1^2 f(x) - g(x)\,\mathrm{d}x &= \int_1^2 -x^2 + 4x + 3 + x^3 - 7x^2 + 10x - 5\,\mathrm{d}x \\
&= \int_1^2 x^3 - 8x^2 + 14x - 2\,\mathrm{d}x \\
&= \frac{1}{4}x^4 - \frac{8}{3}x^3 + 7x^2 - 2x\Big|_1^2 \\
&= \left(4 - \frac{64}{3} + 28 - 4\right) \;-\; \left(\frac{1}{4} - \frac{8}{3} + 7 - 2\right) \\
&= 4\frac{1}{12} \ .
\end{aligned}$$

Thus the required area is $4\frac{1}{12}$.

(a) Which of the following words does *not* describe the functions f and g in the text above?
- (A) Monotonic.
- (B) Polynomial.
- (C) Integrable.
- (D) Differentiable.

(b) Consider the phrases below as replacements for "show f and g respectively" in the text above. Three of them are reasonable replacements. Which one is not?
- (A) "respectively show f and g"
- (B) "show the functions separately"
- (C) "show f and g"
- (D) "respectively show the functions"

continued on the next page

(c) $\int_1^2 f(x)$x̣ is the area under f over the interval $1 \leq x \leq 2$. Which of the following best explains the reason for this.

(A) The fundamental theorem of algebra.
(B) f is a quadratic function.
(C) The fundamental theorem of calculus.
(D) The mathematical definition of the definite integral.

(d) What is the mathematical justification for equation (1)?

(A) Integration is a linear operation.
(B) It follows from the product rule.
(C) Because every continuous function is integrable.
(D) It follows from the chain rule.

(e) What kind of integral is $\int_1^2 f(x) - g(x)$ x̣?

(A) A definite integral.
(B) An indefinite integral.
(C) An antiderivative.
(D) A complex integral.

(f) Why do we compute $\int_1^2 f(x) - g(x)$ x̣ rather than calculating $\int_1^2 f(x)$ x̣ $-$ $\int_1^2 g(x)$ x̣ directly?

(A) So that the integral is in normal form.
(B) Our calculation requires less work because only one integral is involved.
(C) Because we know how to integrate polynomials.
(D) Because of the fundamental theorem of calculus.

(g) How does the term $-\frac{8}{3}x^3$ arise in the third line of the main calculation?

(A) It is the reciprocal of $-8x^2$.
(B) It is the derivative of $-8x^2$.
(C) It is the definite integral of $-8x^2$.
(D) It is an antiderivative of $-8x^2$.

(h) The shaded region of the middle graph has an area of $6\frac{2}{3}$. Calculate the shaded area of the right-hand graph.

(A) $2\frac{7}{12}$
(B) $2\frac{11}{12}$
(C) $2\frac{3}{4}$
(D) $2\frac{5}{6}$