Friedmannove enačbe

- (iz Newtonovega zakona)
- energijski zakon

$$\frac{1}{2}m\dot{R}^2 - \frac{GMm}{R} = E$$

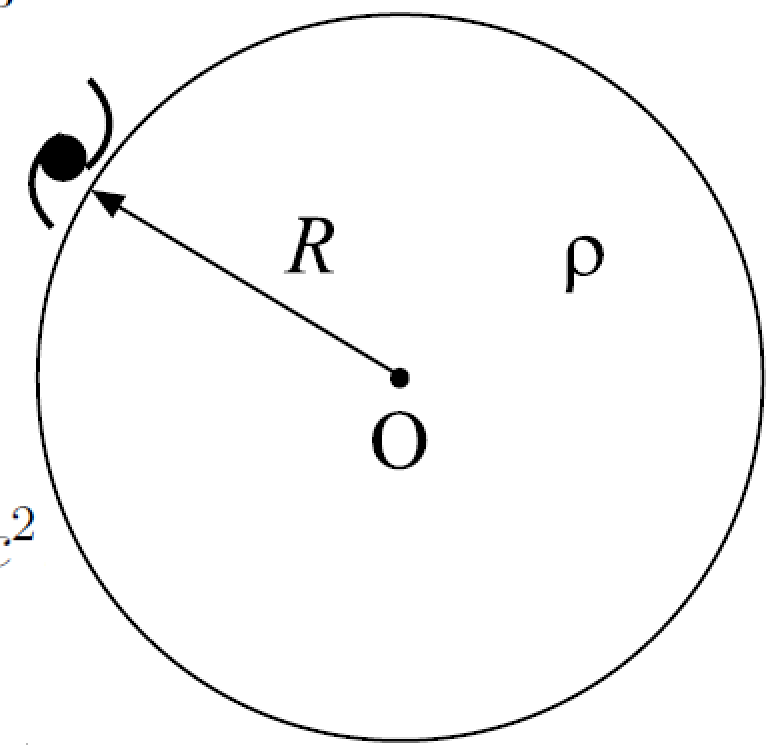
$$M = \frac{4\pi}{3}R^3\rho$$

$$\left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi G}{3}\rho + \frac{2E}{mR^2}$$

$$\frac{2E}{m} = -kc^2$$

$$\boxed{\left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{R^2}}$$

prva Friedmannova enačba



Friedmannove enačbe

- Newtonova enačba gibanja:

$$m\ddot{R} = -\frac{GMm}{R^2} = -\frac{4\pi G}{3}R\rho m$$

dodatni člen s tlakom P

$$\boxed{\frac{\ddot{R}}{R} = -\frac{4\pi G}{3c^2}(\rho c^2 + 3P)}$$

druga Friedmannova enačba

iz prvih dveh enačb -> ni stacionarne rešitve (za primer brez kozmološke konstante)

Friedmannove enačbe

$$\dot{R}^2 = \frac{8\pi G}{3}\rho R^2 - kc^2$$

odvajamo

$$2\dot{R}\ddot{R} = \frac{8\pi G}{3}\rho 2R\dot{R} + \frac{8\pi G}{3}\dot{\rho}R^2$$

uporabimo
drugo F. enačbo

$$2\dot{R}\left[-\frac{4\pi G R}{3c^2}(\rho c^2 + 3P)\right] = \frac{8\pi G}{3}\rho 2R\dot{R} + \frac{8\pi G}{3}\dot{\rho}R^2$$

$$-3\dot{R}(\rho c^2 + P) = \dot{\rho}c^2 R$$

$$\boxed{\dot{\rho}c^2 = -3\frac{\dot{R}}{R}(\rho c^2 + P)}$$

tretja Friedmannova enačba

Enačba stanja P(ρ)

$$\left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{R^2}$$

- prevlada snovi $P \ll \rho c^2$

$$\dot{\rho}c^2 = -3\frac{\dot{R}}{R}(\rho c^2 \cancel{+P})$$

$$\frac{\dot{\rho}}{\rho} = -3\frac{\dot{R}}{R} \Rightarrow \rho \propto R^{-3}$$

Rešitev: $\boxed{R(t) \propto t^{2/3}}$

- prevlada sevanja $P = \frac{1}{3}u = \frac{1}{3}\rho c^2$

$$\dot{\rho}c^2 = -3\frac{\dot{R}}{R}(\rho c^2 + \frac{1}{3}\rho c^2)$$

$$\frac{\dot{\rho}}{\rho} = -4\frac{\dot{R}}{R} \Rightarrow \rho \propto R^{-4}$$

Rešitev: $\boxed{R(t) \propto t^{1/2}}$

v sevalni dobi je širjenje počasnejše kot v dobi, ko prevladuje snov

Ukrivljenost *k*

$$\boxed{\left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{R^2}}$$

- k = 0

$$\left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi G}{3}\rho$$

Kritična gostota:

$$\rho_c = \frac{3H^2}{8\pi G}$$

$$R \propto t^{2/3} \text{ in } \dot{R} \propto t^{-1/3}$$

- k = +1 (zaprto)

$$\frac{8\pi G}{3}\rho = c^2 / R^2$$

$$R = \left(\frac{3c^2}{8\pi G\rho}\right)^{1/2}$$

v določenem trenutku je:

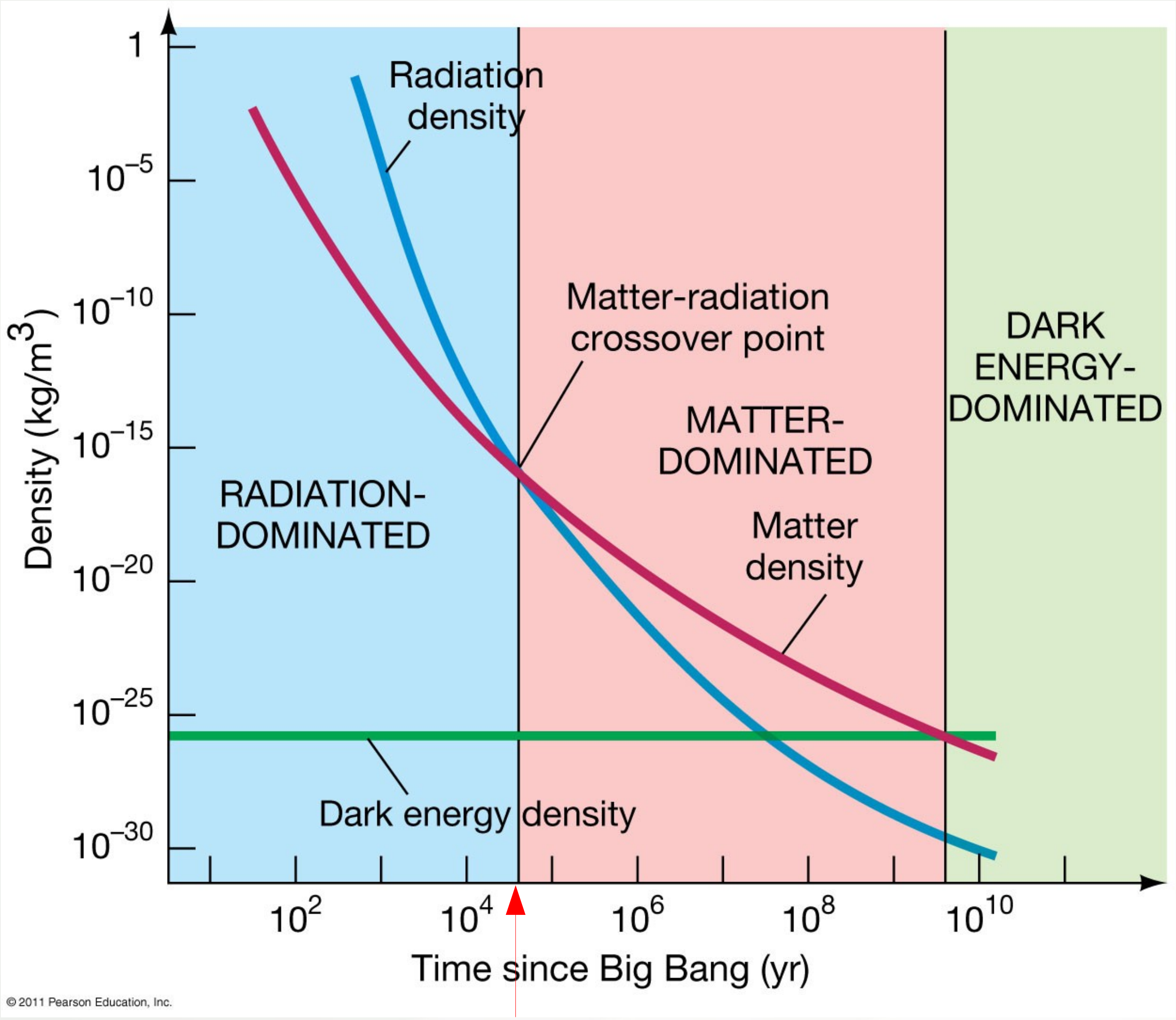
$$\dot{R} = 0$$

- k = -1 (odprto)

Po dovolj dolgem času:

$$\dot{R} = c.$$

Prevlada sevanja/snovi/kozm.konst.



$R = R_0 / 3500$

$t \sim t_0 / 200000$

Ocena starosti vesolja

- $k = 0 \text{ in } \rho_0 = \rho_{0,c}$

$$H^2(t) = \left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi G}{3}\rho = H_0^2 \frac{R_0^3}{R^3}$$

$$\frac{1}{H_0 R_0^{3/2}} \int_0^{R_0} R^{1/2} dR = \int_0^{t_0} dt \rightarrow t_0 = \frac{2}{3H_0}$$

- $\rho = 0$

$$\dot{R} = HR = H_0 R_0$$

$$\frac{1}{H_0 R_0} \int_0^{R_0} dR = \int_0^{t_0} dt \rightarrow t_0 = \frac{1}{H_0}$$

$$\boxed{\frac{2}{3} H_0^{-1} < t_0 < H_0^{-1}}$$

$$H_0 = 70 \text{ km/s/Mpc}$$

$$t_0 = 14 \text{ Gyr}$$

9 do 14 mld let

Friedmannove enačbe (z Λ)

$$\boxed{\left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{R^2} + \frac{\Lambda c^2}{3}}$$

$$\boxed{\frac{\ddot{R}}{R} = -\frac{4\pi G}{3c^2}(\rho c^2 + 3P) + \frac{\Lambda c^2}{3}}$$

$$\boxed{\dot{\rho}c^2 = -3\frac{\dot{R}}{R}(\rho c^2 + P)}$$

- Λ uporabimo za pojasnilo pospešenega širjenja

Kozmološka konstanta Λ

$$\boxed{\left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{R^2} + \frac{\Lambda c^2}{3}} \longrightarrow \frac{8\pi G}{3c^2}\left(\rho c^2 + \frac{c^4}{8\pi G}\Lambda\right)$$

energijska gostota

$$\boxed{\frac{\ddot{R}}{R} = -\frac{4\pi G}{3c^2}(\rho c^2 + 3P) + \frac{\Lambda c^2}{3}} \longrightarrow$$

negativni tlak

obdobje prevlade temne energije (dovolj veliki časi):

$$H^2 = \left(\frac{\dot{R}}{R}\right)^2 \sim \frac{\Lambda c^2}{3}$$

$$R(t) \propto \exp\left(\sqrt{\frac{\Lambda c^2}{3}}t\right) = \exp(Ht)$$

eksponentno širjenje

Kozmološki model k=0, Λ>0

$$\boxed{\left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{R^2} + \frac{\Lambda c^2}{3}}$$

$$\frac{H^2}{H_0^2} = \frac{8\pi G}{3H_0^2}\rho + \frac{\Lambda c^2}{3H_0^2}$$

$$\rho_{c,0} = \frac{3H_0^2}{8\pi G}$$

$$\frac{H^2}{H_0^2} = \frac{\rho}{\rho_{c,0}} + \frac{\Lambda c^2}{3H_0^2}$$

$$\Omega_\Lambda = \frac{\Lambda c^2}{3H_0^2}$$

parameter gostote
za kozmološko konstanto

$$\boxed{\Omega_m + \Omega_\Lambda = 1}$$

H(t) in parametri gostote

- iz prve F. enačbe z uporabo skalirnega faktorja a(t)

$$a(t) = R(t)/R(t_o) = \frac{1}{1+z}$$

izpeljemo Hubblovo konstanto:

$$H(t) \equiv \frac{\dot{R}}{R} = \frac{\dot{a}}{a} = H_o\sqrt{(\Omega_b + \Omega_c)a^{-3} + \Omega_{rad}a^{-4} + \Omega_k a^{-2} + \Omega_\Lambda a^{-3(w+1)}}$$

- $w = -1, \; \Omega_k = 0$

  $\Omega_{rad} \sim 10^{-4}$

$$H(a) = \frac{\dot{a}}{a} = \sqrt{\Omega_m a^{-3} + \Omega_\Lambda}$$

analitična rešitev

$$a(t) = (\Omega_m/\Omega_\Lambda)^{1/3}\sinh^{2/3}(t/t_\Lambda)$$

$$t_\Lambda = \frac{2}{3H_o\sqrt{\Omega_\Lambda}}$$

- kdaj prevlada kozm. konstanta?

$$a(\ddot{a} = 0) = \left(\frac{\Omega_m}{2\Omega_\Lambda}\right)^{1/3}$$

pri a ~ 0,6 in z ~ 0,66 (relativno pred kratkim)

Kozmologija

Osnovni parametri za opis vesolja

Viri: Jones, Lambourne (Chap. 6)

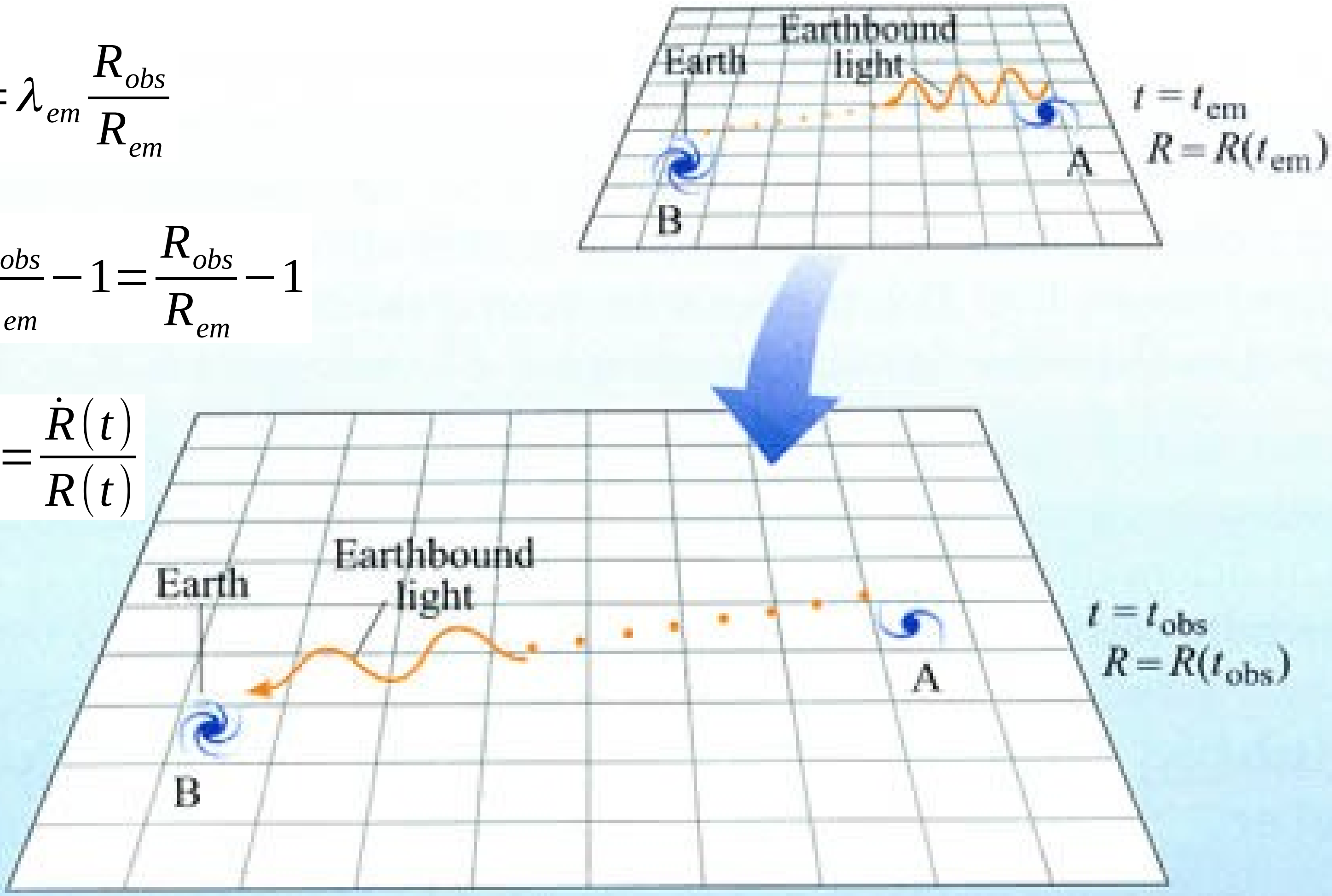
originalne prosojnice prof. Gomboc, dopolnitve in spremembe D. Fabjan

Kozmološki rdeči premik in H(t)

$$\lambda_{obs} = \lambda_{em} \frac{R_{obs}}{R_{em}}$$

$$z = \frac{\lambda_{obs}}{\lambda_{em}} - 1 = \frac{R_{obs}}{R_{em}} - 1$$

$$H(t) = \frac{\dot{R}(t)}{R(t)}$$



Hubblov parameter H(t) predstavlja hitrost spreminjanja R(t) glede na vrednost ob času t

Sistematična odstopanja od Hubblovega zakona

- parameter pojemka q_0

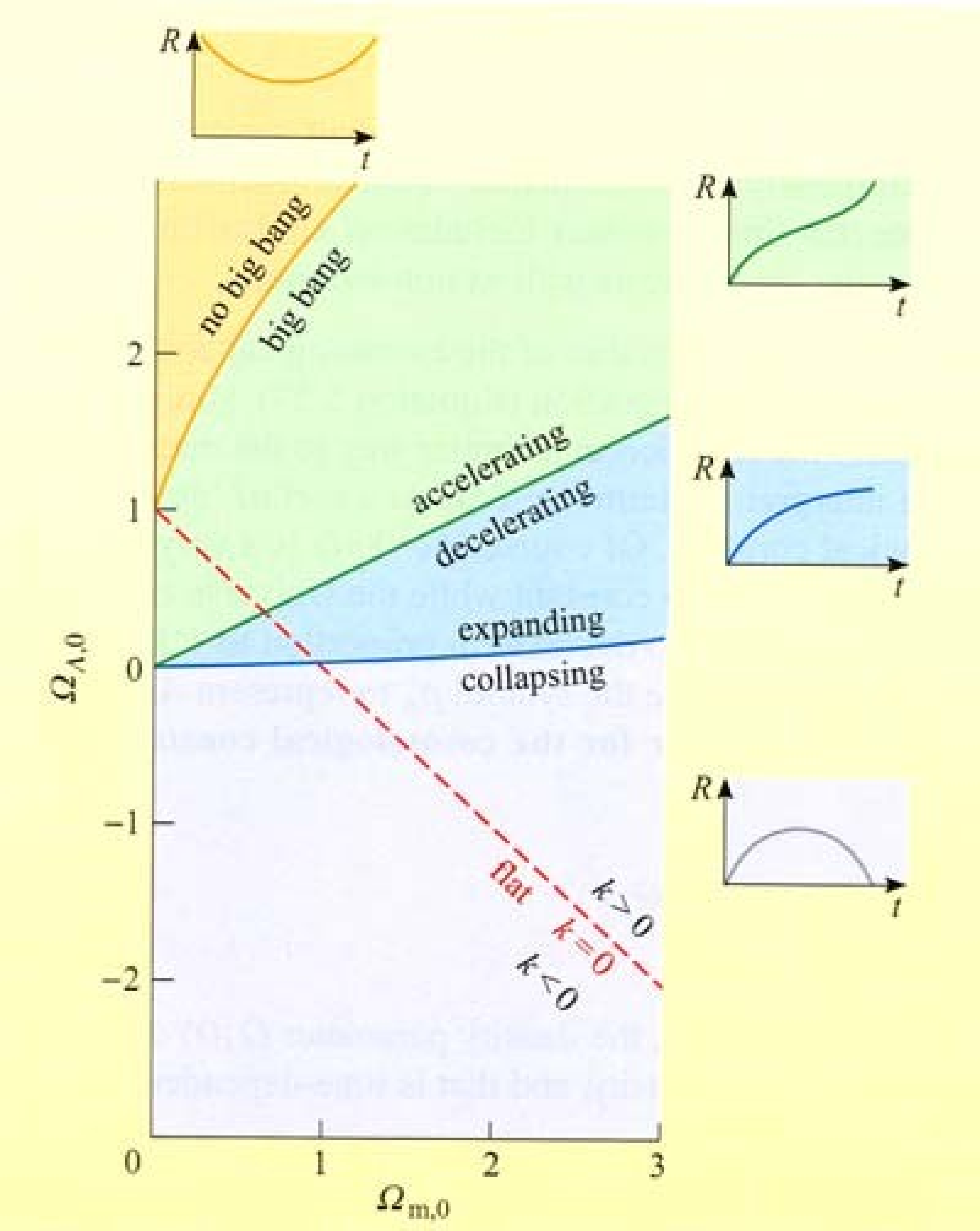
$$q(t) = \frac{-R(t)}{\dot{R}(t)^2} \ddot{R}(t)$$

- odstopanja Hubblovega zakona na večjih oddaljenostih

$$d = \frac{cz}{H_0}$$

$$d = \frac{cz}{H_0} \left[1 + \frac{1}{2} (1 - q_0) z \right]$$

Kritična gostota in gostotni parametri



$$\rho_c = \frac{3H(t)^2}{8\pi G}$$

$$\rho_c(t_0) \sim 10^{-26}\, kg\, m^{-3}$$

$$\Omega_m = \frac{\rho(t)}{\rho_c(t)}$$

$$\Omega_\Lambda = \frac{\rho_\Lambda(t)}{\rho_c(t)} \qquad \rho_\Lambda = \frac{\Lambda c^2}{8\pi G}$$

$$\Omega_m + \Omega_\Lambda - 1 = \frac{k c^2}{R^2 H^2}$$

$$q(t) = \frac{\Omega_m}{2} - \Omega_\Lambda$$