PRAPOK | prevlada sevanja $R(t) \propto t^{1/2}$, $\rho(t) \propto \frac{1}{R^4}$ | prevlada snovi $R(t) \propto t^{2/3}$, $\rho(t) \propto \frac{1}{R^3}$ | odvisno od k RAZLIČNO

! brez kozmološke konstante

$t=0$ — $t=?$ — t_0 — t

$\underbrace{\qquad\qquad\qquad}_{\text{zgodnje obdobje vesolja}}$

$t=?$ prehod med prevlado sevanja in prevlada snovi

CMB = cosmic microwave background → PRASEVANJE $\quad T_{CMB} = 2{,}73\,K$

$$\rho_{rad,0} c^2 = 4{,}2 \cdot 10^{-14} \frac{J}{m^3}$$

$$\rho_{m,0} c^2 = 2{,}6 \cdot 10^{-10} \frac{J}{m^3}$$

$$\rho_\nu = 0{,}68\, \rho_{rad,0}$$

$$\rho_m(t) c^2 = \rho_{m,0}(t_0) c^2 \frac{R(t_0)^3}{R(t)^3}$$

$$\rho_{rad}(t) c^2 = \rho_{rad,0}\, c^2 \frac{R(t_0)^4}{R(t)^4}$$

$$\frac{\rho_m(t)}{\rho_{rad}(t)} = 1 \rightarrow \frac{\rho_{m,0}\, c^2}{1{,}7\, \rho_{rad,0}\, c^2} = \frac{R(t_0)}{R(t)} = 3500 = \frac{t_0^{2/3}}{t^{2/3}}$$

1 (sev) + 0,7 (nevtrini)

$$R(t) = \frac{R(t_0)}{3500}$$

$$t = t_0\,(3500)^{-3/2} = \frac{t_0}{200\,000}$$

Ocenimo starost vesolja

Spodnja meja:

- $k=0$ (ravno vesolje)
- kritična gostota $\rho_{0,cr} = \frac{3H_0^2}{8\pi G}$
- analitična rešitev za prevlado snovi $(\rho \propto R^{-3})$

$$\text{1.F.E.} \quad \left(\frac{\dot R}{R}\right)^2 = \frac{8\pi G}{3}\rho - \cancel{\frac{kc^2}{R^2}}^{\,k=0} = \frac{8\pi G}{3}\frac{H_0^2}{H_0^2}\rho = H_0^2 \frac{\rho}{\rho_{0,cr}} = H_0^2 \left(\frac{R_0}{R}\right)^3 = H^2$$

$$\left(\frac{\dot R}{R}\right)^2 = H_0^2 \left(\frac{R_0}{R}\right)^3 \longrightarrow \frac{\dot R}{R} = H_0 \left(\frac{R_0}{R}\right)^{3/2}$$

$$\frac{\dot{R}}{R} = H_0 \left(\frac{R_0}{R}\right)^{3/2} \quad \Big| \cdot R$$

$$\frac{dR}{dt} = H_0 R_0^{3/2} R^{-1/2}$$

$$\int_0^{t_0} dt = \frac{1}{H_0 R_0^{3/2}} \int_0^{R_0} R^{1/2} dR$$

$$t_0 = \frac{1}{H_0 \cancel{R_0^{3/2}}} \frac{2}{3} \cancel{R_0^{3/2}} = \frac{2}{3H_0}$$

Spodnja meja

2.F.E.:

$$\frac{\ddot{R}}{R} = -\frac{4}{3}\frac{\pi G}{c^2}(\rho c^2 + 3p)$$

Zgornja meja za starost vesolja:

- $\rho = 0$ (prazno vesolje)

2.F.E. $\ddot{R} = 0$ torej $\dot{R} = \text{konst} = H \cdot R = H_0 R_0$

$$\frac{dR}{dt} = H_0 R_0$$

$$\int_0^{t_0} dt = \frac{1}{H_0 R_0} \int_0^{R_0} dR \longrightarrow t_0 = \frac{1}{H_0}$$

zgornja meja

Pokazali smo, da za $0 < \Omega_{m,0} < 1$ je starost vesolja

$$\frac{2}{3H_0} < t_0 < \frac{1}{H_0}$$

($H_0 = 70$ km/s/Mpc) $\quad 9\,\text{Gyr} < t_0 < 14\,\text{Gyr}$

$$t = \frac{t_0}{200000} \sim 65000 \text{ let}$$

FRIEDMANNOVE ENAČBE S KOZMOLOŠKO KONSTANTO

$$\left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{R^2} + \frac{\Lambda c^2}{3} \qquad 1.\text{F.E.}$$

Einstein: statična rešitev: $\dot{R} = \ddot{R} = 0$

$$\frac{\ddot{R}}{R} = -\frac{4\pi G}{3c^2}\left(\rho c^2 + 3p\right) + \frac{\Lambda c^2}{3} \qquad 2.\text{F.E.}$$

$$\dot{\rho} c^2 = -3\frac{\dot{R}}{R}\left(\rho c^2 + p\right) \qquad 3.\text{F.E.}$$

v 1.F.E $\quad \frac{8\pi G}{3}\rho + \frac{\Lambda c^2}{3} = \frac{8\pi G}{3c^2}\left(\rho c^2 + \frac{\Lambda c^4}{8\pi G}\right)$

Kaj se dogaja do zelo velikih časih?

$$\left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi G}{3}\underbrace{\rho}_{\propto R^{-3}} - \underbrace{\frac{kc^2}{R^2}}_{\propto R^2} + \frac{\Lambda c^2}{3} \Rightarrow H^2 = \left(\frac{\dot{R}}{R}\right)^2 \sim \frac{\Lambda c^2}{3}$$

$$\dot{R} \sim \sqrt{\frac{\Lambda c^2}{3}}\, R$$

$$\frac{dR}{R} \sim \sqrt{\frac{\Lambda c^2}{3}}\, dt \longrightarrow R(t) \propto e^{\sqrt{\frac{\Lambda c^2}{3}}\, t} = e^{Ht}$$

REŠITEV ZA ŠIRJENJE VESOLJA S TEMNO ENERGIJO

Opazovanja : $\Lambda > 0$ (konst. vrednost)

$k = 0$

$$\left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi G}{3}\rho - \cancel{\frac{kc^2}{R^2}} + \frac{\Lambda c^2}{3}$$

$$H^2 = \frac{8\pi G}{3}\rho + \frac{\Lambda c^2}{3} \qquad \Big| : H_0^2$$

$$\frac{H^2}{H_0^2} = \frac{8\pi G}{3H_0^2}\rho + \frac{\Lambda c^2}{3H_0^2}$$

$$\rho_{cr,0} = \frac{3H_0^2}{8\pi G}$$

meriter

opazovanja!

$$\frac{H^2}{H_0^2} = \frac{\rho}{\rho_{cr,0}} + \frac{\Lambda c^2}{3H_0^2}$$

$\frac{\Lambda c^2}{3H_0^2} = \Omega_\Lambda$ gostotni parameter za kozmološko konst.

parameter gostote $\Omega_m = \frac{\rho_m}{\rho_{cr}}$

m = mater

$$\frac{H^2}{H_0^2} = \Omega_m + \Omega_\Lambda$$

ob $t = t_0 \longrightarrow \Omega_{m,0} + \Omega_{\Lambda,0} = 1$

- če je $k = 0$ (ravno vesolje) velja ob kateremkoli času $\Omega_m + \Omega_\Lambda = 1$!
- če je $k = 0$ in $\Lambda = 0$ $\quad \Omega_m = 1$!
- če je $\Lambda > 0$ in k poljuben $(k \neq 0)$:

$\Omega_m + \Omega_\Lambda > 1$ zaprto vesolje

$\Omega_m + \Omega_\Lambda < 1$ odprto vesolje

pregledam snov v vesolju

$$\Omega_m = \frac{\rho_{m,0}}{\rho_{cr,0}} = 1$$

H_0 rdeči premik galaksij

$$R \propto \frac{1}{1+z} \qquad \left(\text{glej prosojnico: } z = \frac{R_0}{R} - 1 \rightarrow \frac{R}{R_0} = \frac{1}{1+z}\right)$$

$$R(t_0) \propto \frac{1}{1+0}$$

$$\frac{R}{R_0} = \frac{1}{1+z}$$

$$\left.\begin{array}{l} R(t_1) \propto \dfrac{1}{1+z(t_1)} \\ R(t_2) \propto \dfrac{1}{1+z(t_2)} \end{array}\right\} \frac{R(t_1)}{R(t_2)} = \frac{1+z(t_2)}{1+z(t_1)}$$

1\. F. E. z $\frac{R}{R_0} = \frac{1}{1+z}$ lahko zapišemo kot

$a \to 0 \quad z \to \infty$

$a = 1 \quad z = 0$

skalirni faktor

$$H = \frac{\dot{R}}{R} = \frac{\dot{a}}{a} =$$

$$= H_0 \sqrt{(\Omega_b + \Omega_c)\, a^{-3} + \Omega_{rad}\, a^{-4} + \Omega_k a^{-2} + \underbrace{\Omega_\Lambda a^{-3(w+1)}}}$$

barionska snov — hladna temna snov

$$\Omega_m = \frac{\rho}{\rho_{cr}}$$

$p_\Lambda = w \rho_\Lambda c^2$

$w = -1$ za kozm. konst.

$w \neq -1$ krintesenca

Rešitev v primeru:

- $k=0,\ \Lambda > 0\ (w=-1)$, Ω_{rad} zanemarimo:

$$H(a) = \frac{\dot{a}}{a} = \sqrt{\Omega_m a^{-3} + \Omega_\Lambda}$$

$$a(t) = \left(\frac{\Omega_m}{\Omega_\Lambda}\right)^{1/3} \sinh^{2/3}\left(\frac{t}{t_\Lambda}\right) \quad ; \quad t_\Lambda = \frac{2}{3H_0\sqrt{\Omega_\Lambda}}$$

(uporabna za $a > 0{,}01$ ($t > 10^7$ let))

od kdaj se tako vesolje širi pospešeno?

$$a(\ddot{a}=0) = \left(\frac{\Omega_m}{2\Omega_\Lambda}\right)^{1/3}$$

$a \sim 0{,}6$

$z \sim 0{,}66$

OPAZOVALNI PREIZKUSI

Glavna opazovanja, ki določajo kozmološke parametre, so:

- obstoj in struktura prasevanja
- porazdelitev snovi na velikih skalah
- kemijska zastopanost vodika, helija in litija ← prvinska nukleosinteza
- pospešeno širjenje vesolja z oddaljenimi supernovami Ia

Kozmološki rdeči premik

TRW metrika: za fotone $ds = 0$

$$0 = c^2 dt^2 - R^2(t) \frac{dr^2}{1-kr^2} \qquad (\theta = \phi = 0)$$

@ r_e $\rightsquigarrow t_e$ $\rightsquigarrow t_e + \Delta t_e$ | $r = 0$ t_0 $t_0 + \Delta t_0$

prvi foton
$$\int_{t_e + \Delta t_e}^{t_0 + \Delta t_0} \frac{dt}{R(t)} = \frac{1}{c} \int_0^{r_e} \frac{dr}{\sqrt{1-kr^2}}$$

drugi foton

$$\int_{t_e+\Delta t_e}^{t_0+\Delta t_0} \frac{dt}{R(t)} - \int_{t_e}^{t_0} \frac{dt}{R(t)} = 0$$

$$\int_{t_e}^{t_0} - \int_{t_e}^{t_e+\Delta t_e} + \int_{t_0}^{t_0+\Delta t_0} - \int_{t_e}^{t_0} \frac{dt}{R(t)} = 0$$

$$\frac{\Delta t_e}{R(t_e)} = \frac{\Delta t_0}{R(t_0)} \qquad \Delta t_e = \frac{\lambda_e}{c} \qquad \Delta t_0 = \frac{\lambda_0}{c}$$

$$\frac{\Delta t_0}{\Delta t_e} = \frac{\lambda_0}{\lambda_e} = \frac{R(t_0)}{R(t_e)} = 1 + z$$

Razdalje:

- gostota svetlobnega toka

$$j = \frac{L}{4\pi d_L^2} = \frac{L}{4\pi d^2 (1+z)^2}$$

razdalja izsevа d_L

$$d_L = d(1+z) \underset{k=0}{=} a \cdot r\,(1+z)$$

- sprememba en. fotonov
- fotoni prihajajo manj pogosto

- kotna velikost objekta

$$l = r_0 a(t_e) \vartheta$$

$$\theta = \frac{l}{r_0 a(t_e)} = \frac{l(1+z)}{r_0 a_0}$$

razdalja kotnega premera d_{prem}

$$d_{prem} = \frac{l}{\sin\vartheta} \sim \frac{l}{\theta} = \frac{r_0 a_0}{1+z} = \frac{d_L}{(1+z)^2}$$

$$d_{prem} = \frac{d}{1+z} = \frac{d_L}{(1+z)^2}$$

Mikrovalovno sevanje ozadja

$$\left.\begin{aligned}\rho_{rad} &\propto \frac{1}{a^4}\\ \rho_{rad} &= \left(\frac{4}{c}\right)\sigma T^4\end{aligned}\right\} \quad T \propto \frac{1}{a}$$

$T(t_0) = 2{,}73\,K$

$T \sim 3000\,K$