Jedra, kvarki, leptoni

1. kolokvij, 9.december 2021

(Možnih je 4.5 od 4.0 točk.)

1. *(1.25 točke)* Obravnavamo razpade mezona Δ^+ iz barionskega dekupleta, ki razpada preko močne interakcije v ΣK (barion iz okteta in kaon).

   (a) Uporabi izospinsko simetrijo za napoved razmerja razpadnih širin

   $$\frac{\Gamma(\Delta^+ \to \Sigma^0 K^+)}{\Gamma(\Delta^+ \to \Sigma^+ K^0)}\,.$$

   Upoštevaj, da imata oba bariona Σ enako maso, kot tudi mezona K.

   (b) Napovej razmerje magnetnih momentov $\mu_{\Sigma^+}/\mu_{\Sigma^-}$, če velja za magnetne momente kvarkov $\mu_u = -2\mu_d = -10/3\mu_s$.

2. *(1.0 točke)* Obravnavaj izmenjavo enega gluona med kvarkom q in antikvarkom $\bar{q}$, ki se sipata iz barvnega stanja $1/\sqrt{2}(R\bar{R}-G\bar{G})$ v enako končno stanje. Poišči barvni faktor, ki ga da vsota amplitud za vse možne gluone med q in $\bar{q}$, in pokaži, da je pozitiven, da je torej nerelativističen potencial odbojen. (Ne pozabi upoštevati, da imata q in $\bar{q}$ nasprotno sklopitveno konstanto g_s).

3. *(1.0 točk)* Diracova enačba.

   a) Pokaži, da se Diracov bilinear $T^{\mu\nu} = \bar{\psi}\sigma^{\mu\nu}\psi$ pri transformaciji prostorske parnosti transformira kot običajen Lorentzov tenzor:

   $$T^{0i} \to -T^{0i},$$
   $$T^{ij} \to +T^{ij},$$

   kjer sta $i, j = 1, 2, 3$.

   b) Izračunaj $\gamma^\mu \not{p} \not{k} \gamma_\mu$.

4. *(1.25 točke)* Na protonu sipamo elektrone, da bi se kaj naučili o porazdelitvi naboja v protonu. Najprej uporabimo nepolarizirane vhodne elektrone z nizko energijo, $E = 20\,\mathrm{MeV}$, zato lahko proton obravnavaš kot fiksno tarčo. Merimo diferencialni sipalni presek pomnožen s $\sin^4\vartheta/2$, $z(\vartheta) \equiv \sin^4(\vartheta/2)(d\sigma/d\Omega)$. Izmerimo, da je razmerje $z(6^\circ)/z(5^\circ) = 1 - 0.00881$. Iz meritve oceni naklon oblikovne funkcije za proton $dF(q^2)/dq^2$ pri $q^2 = 0$. Namig: zapiši $z(6^\circ) \approx z(5^\circ) + (dz/d\vartheta)\Delta\vartheta$. Upoštevaj, da je q^2 v obeh primerih ≈ 0, odvod $F(q^2)$ aproksimiraj z diferenco! Iz dobljenega odvoda F izračunaj še nabojni radij protona, $\langle r^2 \rangle$, ki je definiran kot povprečen kvadrat radija nabojne porazdelitve.

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(1.) a) $R = \frac{\Gamma(\Delta^+ \to \Sigma^0 K^+)}{\Gamma(\Delta^+ \to \Sigma^+ K^0)} \propto \left| \frac{(\langle 1\,0 | \langle \frac{1}{2}\,\frac{1}{2} |) | \frac{3}{2}\,\frac{1}{2} \rangle}{(\langle 1\,1 | \langle \frac{1}{2}\,-\frac{1}{2} |) | \frac{3}{2}\,\frac{1}{2} \rangle} \right|^2$ 1/4

$|\frac{3}{2}\,\frac{1}{2}\rangle = \frac{1}{\sqrt{3}} |1\,1\rangle |\frac{1}{2}\,-\frac{1}{2}\rangle + \sqrt{\frac{2}{3}} |1\,0\rangle |\frac{1}{2}\,\frac{1}{2}\rangle$

1/4

$\Rightarrow R = 2$ +

b) $\Sigma^+(\uparrow) = \frac{1}{\sqrt{18}} \Big(uus\ (2\uparrow\uparrow\downarrow - \downarrow\uparrow\uparrow - \uparrow\downarrow\uparrow)$

$+ \text{2 permutacji} \Big)$ +

$\Sigma^-(\uparrow)$ isto, le $u \leftrightarrow d$ +

$\mu_{\Sigma^+} = \frac{3}{18}\left(2^2(2\mu_u - \mu_s) + (-1)^2\mu_s + (-1)^2\mu_s\right)$

$= \frac{1}{6}(8\mu_u - 2\mu_s)$ 1/4

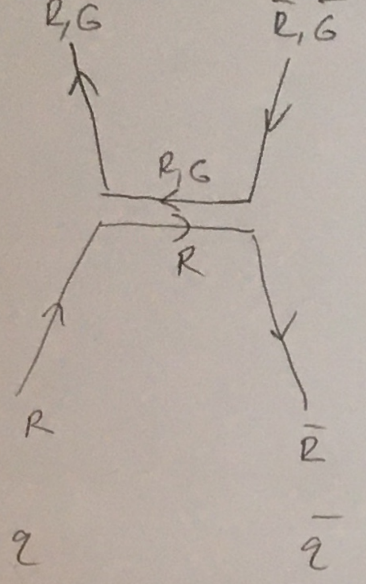
$\mu_{\Sigma^-} = \frac{1}{6}(8\mu_d - 2\mu_s)$

$\frac{\mu_{\Sigma^+}}{\mu_{\Sigma^-}} = \frac{4\mu_u - \mu_s}{4\mu_d - \mu_s} = \frac{4\mu_u + \frac{3}{10}\mu_u}{-2\mu_u + \frac{3}{10}\mu_u} = \frac{-43}{17}$ +

+

+

2.



barvni faktor

$R\bar R \to R\bar R$: $\left(\frac{1}{\sqrt2}\right)^2\left(\frac{1}{\sqrt2}\left(-\frac{1}{\sqrt2}\right)+\frac{1}{\sqrt6}\left(-\frac{1}{\sqrt6}\right)\right)$

$= -\frac12\left(\frac12+\frac16\right) = -\frac13$ 1/4+

$R\bar R \to G\bar G$: $\frac{1}{\sqrt2}\left(-\frac{1}{\sqrt2}\right)\left(1(-1)\right) = \frac12$ 1/4+

in še dva prispevka od $G\bar G \to G\bar G$

$G\bar G \to R\bar R$

$\frac{1}{\sqrt2}(R\bar R - G\bar G)$

skupaj $2\cdot\left(\frac12-\frac13\right) = \underline{\underline{\frac13}} > 0$

1/4

gluoni: $\{R\bar B, G\bar B, R\bar G, G\bar R, B\bar R, B\bar G$

$\frac{1}{\sqrt2}(B\bar R - G\bar G), \frac{1}{\sqrt6}(R\bar R + G\bar G - 2B\bar B)\}$

3. a) $\bar\psi\,\sigma^{\mu\nu}\psi \longmapsto \overline{\gamma^0\psi}\,\sigma^{\mu\nu}\gamma^0\psi$

$= \psi^\dagger\gamma^0\gamma^0\sigma^{\mu\nu}\gamma^0\psi$

$= \bar\psi\underbrace{(\gamma^0\sigma^{\mu\nu}\gamma^0)}_{(\sigma^{\mu\nu})^\dagger}\psi$ } 1/4

T^{0i} : ~~$\gamma^0\frac{i}{2}[\gamma^0,\gamma^i]\gamma^0 = \frac{i}{2}$~~

$(\sigma^{0i})^\dagger = \left(\frac{i}{2}[\gamma^0,\gamma^i]\right)^\dagger = -\frac{i}{2}[\gamma^{i\dagger},\gamma^{0\dagger}]$

$= -\frac{i}{2}[-\gamma^i,\gamma^0] = -\frac{i}{2}[\gamma^0,\gamma^i] = -\sigma^{0i}$ +

T^{ij} : $(\sigma^{ij})^\dagger = -\frac{i}{2}[-\gamma^j,-\gamma^i] = +\frac{i}{2}[\gamma^i,\gamma^j] = \sigma^{ij}$ +

b) $\gamma^\mu \not p \not k \gamma_\mu = \gamma^\mu \not p\,(2k_\mu - \gamma_\mu \not k) = 2\not k\not p - \underbrace{\gamma^\mu\not p\gamma_\mu}_{-2\not p}\not k$ + +

$= 2(\not k\not p + \not p\not k) = 2k_\mu p_\nu\{\gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu\}$

$= 4k \cdot p$ +

(4.) $E = 20\ MeV$

$\vartheta = 5°$

$\Delta\vartheta = 1° \rightarrow$ bolje v radianih $\Delta\vartheta = \frac{\pi}{180}$

$$\frac{z(\vartheta + \Delta\vartheta)}{z(\vartheta)} = 1 - x \ ; \quad x = 0{,}00881,$$

kjer $z(\vartheta) = \sin^4\frac{\vartheta}{2}\ \frac{d\sigma}{d\Omega}$.

$$\frac{d\sigma}{d\Omega} = \frac{(Z\alpha)^2 E^2}{4\vec{k}^4 \sin^4\frac{\vartheta}{2}}\left(1 - v^2\sin^2\frac{\vartheta}{2}\right) F(\vec{q})^2 \quad +$$

$|\vec{k}| \approx E, \quad v \approx 1, \quad Z = 1$

$$z(\vartheta) = \frac{\alpha^2}{4E^2}\cos^2\frac{\vartheta}{2}\ F(\vec{q})^2 \ + ; \qquad F(\vec{q}) = F(\vec{q}^2)$$

$$\frac{z(\vartheta+\Delta\vartheta)}{z(\vartheta)} = \frac{z(\vartheta) + \Delta\vartheta\frac{dz}{d\vartheta}}{z(\vartheta)} = 1 + \Delta\vartheta\,\frac{\frac{dz}{d\vartheta}}{z(\vartheta)} \quad 1/4$$

$$= 1 + \Delta\vartheta\,\frac{-\cancel{\frac{1}{2}}\cos\frac{\vartheta}{2}\sin\frac{\vartheta}{2}\,F(q^2)^2 + 2F(q^2)\frac{dF}{dq^2}\frac{dq^2}{d\vartheta}\cos^2\frac{\vartheta}{2}}{\cos^2\frac{\vartheta}{2}\ F(q^2)^2}$$

$$\Rightarrow \Delta\vartheta\,\frac{-\sin\frac{\vartheta}{2}F(q^2) + 2\frac{dF}{dq^2}\frac{dq^2}{d\vartheta}\cos\frac{\vartheta}{2}}{\cos\frac{\vartheta}{2}\ F(q^2)} = -x$$

$q^2 = 4k^2\sin^2\frac{\vartheta}{2}, \quad \frac{dq^2}{d\vartheta} = 2k^2\sin\vartheta \quad 1/4$

Poenostavitve: $q^2 \ll M^2$ $\quad q^2 \approx 0, \quad F(q^2) \approx 1, \quad \cancel{\cos\frac{\vartheta}{2} \approx 1}$ +

$\cancel{-\sin\frac{\vartheta}{2} + 2\frac{dF}{dq^2}}$

$$\Delta\vartheta\left(-\operatorname{tg}\frac{\vartheta}{2} + 2\frac{dF}{dq^2}\cdot 2E^2\sin\vartheta\right) = -x$$

$$\frac{dF}{dq^2} = \frac{\operatorname{tg}\frac{\vartheta}{2} - \frac{x}{\Delta\vartheta}}{4E^2\sin\vartheta} = -0{,}0033\ \mathrm{MeV}^{-2}$$ 1/4

$$F(q^2) = 1 - \frac{1}{6}\langle r^2\rangle q^2 + \dots$$

$$\langle r^2\rangle = -6\frac{dF}{dq^2} = 0{,}0198\ \mathrm{MeV}^{-2}$$

$$\langle r^2\rangle_{\text{SI enote}} = 0{,}0198\ \mathrm{MeV}^{-2}(\hbar c)^2$$

$$= 0{,}0198\cdot\mathrm{MeV}^{-2}(197\ \mathrm{MeV\ fm})^2$$

$$= 768\ \mathrm{fm}^2$$ 1/4