1.

$$\frac{d\sigma}{dE'\,d\Omega} = \frac{4\alpha^2 E'^2}{Q^4}\left\{ W_2 \cos^2\frac{\theta}{2} + 2W_1 \sin^2\frac{\theta}{2}\right\}$$

$W_2 \uparrow \quad \frac{F_2(x)}{\nu}$ $\qquad$ $2W_1 \uparrow \quad \frac{F_1(x)}{M} = \frac{F_2(x)}{2Mx}$

$E = 5\ \text{GeV}$
$\theta = 30°$
$x = 0{,}5$

$$= \frac{4\alpha^2 E'^2}{Q^4 \nu} F_2(x)\left\{\cos^2\frac{\theta}{2} + \frac{\nu}{Mx}\sin^2\frac{\theta}{2}\right\}$$

$$x = \frac{Q^2}{2M\nu} = \frac{2 \cdot 4 E E' \sin^2\frac{\theta}{2}}{2M(E-E')}$$

$$E' = \frac{MxE}{2E\sin^2\frac{\theta}{2} + Mx} = \frac{E}{1 + \frac{2E}{Mx}\sin^2\frac{\theta}{2}} = \frac{5\ \text{GeV}}{1 + \frac{10 \cdot \sin^2 15°}{0{,}5 \cdot 0{,}94}} = 2{,}06\ \text{GeV} \qquad 1/4$$

$$\frac{d\sigma^p}{d\sigma^n} = \frac{F_2^p(x)}{F_2^n(x)} = \frac{4/9\, u_v + \frac{1}{9} d_v}{\frac{4}{9} d_v + \frac{1}{9} u_v} = \frac{4\frac{u_v}{d_v} + 1}{4 + \frac{u_v}{d_v}}$$

$$= \frac{4 \cdot 2 + 1}{4 + 2} = \frac{3}{2} \qquad 1/2$$

$$\left.\frac{d\sigma^p}{d\sigma^n}\right|_{x\to 0} = \left.\frac{F_2^p(x)}{F_2^n(x)}\right|_{x\to 0} \approx \frac{4/3\, S(x)}{4/3\, S(x)} \approx 1 \qquad 1/4$$

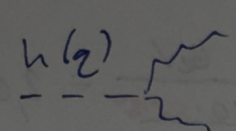
(2.) $D_\mu = \partial_\mu + \frac{ig}{c_W}(T_3 - Q s_W^2) Z_\mu$

$\mathcal{L} = |D_\mu H|^2 = (D_\mu H)^+ (D^\mu H)$

$D_\mu H = \partial_\mu \begin{pmatrix} 0 \\ \frac{v+h}{\sqrt{2}} \end{pmatrix} + \frac{ig}{c_W}\left(-\frac{1}{2}\right)\begin{pmatrix} 0 \\ \frac{v+h}{\sqrt{2}} \end{pmatrix} Z_\mu$

$|D_\mu H|^2 = \frac{g^2}{4c_W^2}\left(\frac{v+h}{\sqrt{2}}\right)^2 Z_\mu Z^\mu + \dots$

$= \frac{g^2 h^2}{8 c_W^2} Z_\mu Z^\mu + \underbrace{\frac{g^2 v h}{4 c_W^2} Z_\mu Z^\mu} + \dots$



$= 2! \cdot \frac{ig^2 v}{4c_W^2} g^{\mu\nu}$ 1/2

2! ta faktor pride zaradi 2! permutacij polja Z_μ (se ne odbija točk)

$i\mathcal{M} = \frac{ig^2 v}{2c_W^2} \varepsilon^*(p) \cdot \varepsilon'^*(p') +$

$\overline{|\mathcal{M}|^2} = \frac{g^4 v^2}{4c_W^4} \sum_{\varepsilon,\varepsilon'} \varepsilon^{\mu\nu} \varepsilon_\nu \varepsilon'^\nu_\mu \varepsilon'_\nu$

$= \frac{g^4 v^2}{4c_W^4}\left(-\eta^{\mu\nu} + \frac{p^\mu p^\nu}{m_Z^2}\right)\left(-\eta_{\mu\nu} + \frac{p'_\mu p'_\nu}{m_Z^2}\right)$

$= \frac{g^4 v^2}{4c_W^4}\left(4 - \frac{p^2}{m_Z^2} - \frac{p'^2}{m_Z^2} + \frac{(p\cdot p')^2}{m_Z^4}\right)$ $\quad p\cdot p' = \frac{m_h^2 - 2m_Z^2}{2}$

$= \frac{g^4 v^2}{4c_W^4}\left(2 + \frac{1}{4}\left(\frac{m_h^2}{m_Z^2} - 2\right)^2\right)$

$= \frac{g^4 v^2}{4c_W^4}\left(3 - \frac{m_h^2}{m_Z^2} + \frac{1}{4}\left(\frac{m_h^2}{m_Z^2}\right)^2\right)$ 1/4 $\quad x \equiv \frac{m_h^2}{4m_Z^2} = 1{,}23$

$\Gamma_{h\to ZZ} = \frac{1}{2!}\frac{|\vec{p}|}{8\pi m_h^2}\overline{|\mathcal{M}|^2} = \frac{\sqrt{\left(\frac{m_h}{2}\right)^2 - m_Z^2}}{8\pi m_h^2} \cdot \frac{g^4 v^2}{4c_W^4}\left(3 - 4x + 4x^2\right)$

$= \frac{m_h}{32\pi m_h^2}\sqrt{1 - \frac{1}{x}}\,\frac{g^4 v^2}{4c_W^4}\left(3 - 4x + 4x^2\right)$ 1/4

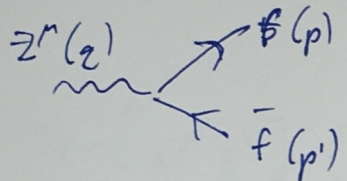
$$m_W^2 = \frac{1}{4} g^2 v^2 \quad \Rightarrow \quad g^4 v^2 = \frac{g^4 v^4}{v^2} = \frac{16 m_W^4}{v^2}$$

$$\Gamma_{h\to ZZ} = \frac{\cancel{8}}{v^2} \frac{m_W^4}{c_W^4} \frac{1}{16\pi m_h} \sqrt{1-\frac{1}{x}} \left(3 - 4x + 4x^2\right)$$

(m_Z^4)

$$= \frac{m_Z^4}{2\pi v^2 m_h} \sqrt{1-\frac{1}{x}} \left(3 - 4x + 4x^2\right) = \cancel{3,4}\ 4{,}5\ \text{GeV} \quad 1/4$$

3. $Z \to b\bar{b}$



$$= \frac{ig}{c_W} \left(g_L^f \gamma^\mu P_L + g_R^f \gamma^\mu P_R\right)$$

$$g_L^f = Q s_W^2 - T^3 \qquad g_L^b = \frac{1}{2} - \frac{1}{3} s_W^2$$

$$g_R^f = Q s_W^2 \qquad g_R^b = -\frac{1}{3} s_W^2 \qquad 1/4$$

$$i\mathcal{M} = \frac{ig}{c_W} \bar{u}(p) \left(g_L^b \gamma^\mu P_L + g_R^b \gamma^\mu P_R\right) v(p') \, \varepsilon_\mu^* \quad 1/4$$

(3 barve)

$$\overline{|\mathcal{M}|^2} = \frac{3}{3} \frac{g^2}{c_W^2} \mathrm{Tr}\Big[(\not{p} + m_b)\left(g_L^b \gamma^\mu P_L + g_R^b \gamma^\mu P_R\right)(\not{p}' - m_b)\left(g_L^{b\nu} \gamma^\nu P_L + g_R^{b\nu} \gamma^\nu P_R\right)\Big]$$

$$\left(-\eta_{\mu\nu} + \frac{q_\mu q_\nu}{m_Z^2}\right) \quad 1/4$$

$3 = 2 \cdot S_q + 1$ $\quad 1/4$

$$= \frac{g^2}{c_W^2} \mathrm{Tr}\Big[\not{p}\left(g_L^b\right)^2 \not{p} \gamma^\mu \not{p}' \gamma^\nu P_L + \left(g_R^b\right)^2 \not{p} \gamma^\mu \not{p}' \gamma^\nu P_R$$

$$+ m_b^2 g_L^b g_R^b \gamma^\mu \gamma^\nu \left(P_L + P_R\right)\Big] \left(-\eta_{\mu\nu} + \frac{q_\mu q_\nu}{m_Z^2}\right) \quad 1/4$$

$$\mathrm{Tr}\left[\not p \gamma^\mu \not p' \gamma^\nu P_L\right]\left(-\eta_{\mu\nu} + \frac{2p\cdot z\, z_\nu}{m_Z^2}\right)$$

$$= +\mathrm{Tr}\left[\not p \not p' P_L\right] + \frac{1}{m_Z^2}\mathrm{Tr}\left[\not p \not z \not p' \not z P_L\right]$$

$$= 4p\cdot p' + \frac{1}{m_Z^2} 2\left(2p\cdot z\, p'\cdot z - z^2 p\cdot p'\right)$$

$$= 2p\cdot p' + \frac{2p\cdot z\; 2\cdot p'\cdot z}{m_Z^2}$$

$$= m_Z^2 - 2m_b^2 + m_Z^2$$

$$= 2\left(m_Z^2 - m_b^2\right)$$

$$q - p = p'\,|^2$$

$$z^2 + p^2 - 2p\cdot z = p'^2$$

$$p\cdot z = \frac{q^2}{2}$$

$$2p\cdot p' = q^2 - 2m_b^2$$

$$= m_Z^2 - 2m_b^2$$

$$\mathrm{Tr}\left[\gamma^\mu \gamma^\nu\right]\left(-\eta_{\mu\nu} + \frac{2p\cdot z\, z_\nu}{m_Z^2}\right)$$

$$= -\mathrm{Tr}[4] + \frac{1}{m_Z^2}\mathrm{Tr}[z^2]$$

$$= -16 + 4 = -12$$

$$\overline{|\mathcal{M}|^2} = \frac{g^2}{c_W^2}\left[2\left(g_L^{b\,2} + g_R^{b\,2}\right)\left(m_Z^2 - m_b^2\right) + 12 m_b^2\, g_L^b g_R^b\right] \cdot 1/4$$

$$\Gamma_{Z\to b\bar b} = \frac{\sqrt{\left(\frac{m_Z}{2}\right)^2 - m_b^2}}{8\pi m_Z^2}\,\frac{g^2 m_Z^2}{c_W^2}\left[2\left(g_L^{b\,2} + g_R^{b\,2}\right)\left(1 - \frac{m_b^2}{m_Z^2}\right) + \frac{12 m_b^2}{m_Z^2} g_L^b g_R^b\right]$$

$$= \frac{m_Z}{16\pi}\frac{g^2}{c_W^2}\sqrt{1 - \frac{4m_b^2}{m_Z^2}}\cdot\Big[\qquad\Big]$$

$$= \frac{m_Z}{16\pi}\frac{g^2\cdot m_Z^2}{m_W^2}\sqrt{1-\kappa}\left[2\left(g_L^{b\,2} + g_R^{b\,2}\right)\left(1 - \frac{\kappa}{4}\right) + 3\kappa\, g_L^b g_R^b\right] \cdot 1$$

$$\frac{\Gamma}{\Gamma_Z} = \frac{g^2}{8m_W^2} = \frac{G_F m_Z^3}{2\sqrt{2}\pi}\sqrt{1-\kappa}\Big[\qquad\Big] = \begin{cases} 0{,}35109 \\ 0{,}35420 \end{cases}$$