Pisni izpit iz Matematike 1

14. 6. 2016

Veliko uspeha!

1. naloga

Poišči kompleksna števila, ki rešijo enačbo

$$z^3 = \frac{1+i}{1-i}.$$

Dana je funkcija

$$f(x) = \frac{4x-12}{(x-2)^2}.$$

Določi definicijsko območje funkcije f, njeno zalogo vrednosti, asimptote, območja padanja oz. naraščanja, lokalne ekstreme, območja konveksnosti oz. konkavnosti, prevoje ter natančno nariši njen graf.

2. naloga

Dana je funkcija

$$f(x) = \begin{cases} (2x+1)^3, & x < -1 \\ a + b\tan\frac{\pi x}{4}, & -1 \leq x \leq 1 \\ 1 + \ln x, & x > 1. \end{cases}$$

Določi števili a in b tako, da bo f zvezna v $x = -1$ in $x = 1$.

3. naloga

(a) Izračunaj integral

$$\int_0^1 x^2 e^{-x} dx.$$

(b) Izračunaj ploščino lika omejenega s krivuljami $y = \sqrt{x}$, $y = -x + 2$, $y = 0$.

1

$$z^3 = \frac{1+i}{1-i} = \frac{(1+i)^2}{2} = i$$ 5

$z = |z|(\cos\varphi + i\sin\varphi)$

$z^3 = |z|^3(\cos 3\varphi + i \sin 3\varphi) = i = \cos\frac{\pi}{2} + i\sin\frac{\pi}{2}$

$\Longrightarrow |z| = 1 \;\&\; \varphi = \frac{\pi}{6} + \frac{2k\pi}{3}\;;\; k = 0, 1, 2$

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$z_1 = \cos\frac{\pi}{6} + i\sin\frac{\pi}{6} = \underline{\underline{\frac{\sqrt{3}}{2} + i\cdot\frac{1}{2}}}$

$z_2 = \cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6} = \underline{\underline{-\frac{\sqrt{3}}{2} + i\cdot\frac{1}{2}}}$

$z_3 = \cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2} = \underline{\underline{-i}}$

2

$f(x) = \frac{4(x-3)}{(x-2)^2}$ Ničla: $x = 3$

Pol: $x = 2$ (dvojni)

5

$D_f = \mathbb{R} \setminus \{2\}$ Asimptota: $y = 0$

$$f'(x) = 4\,\frac{(x-2)^{\cancel{2}} - 2(x-3)\cancel{(x-2)}}{(x-2)^{\cancel{4}\,3}} =$$

$$= 4\,\frac{x-2-2x+6}{(x-2)^3} = 4\,\frac{4-x}{(x-2)^3}$$

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$x < 2: f'(x) < 0$ (pada)

$2 < x < 4: f'(x) > 0$ (raste)

$x > 4: f'(x) < 0$ (pada)

v $x = 4$ je max

$T_1(4, 1)$

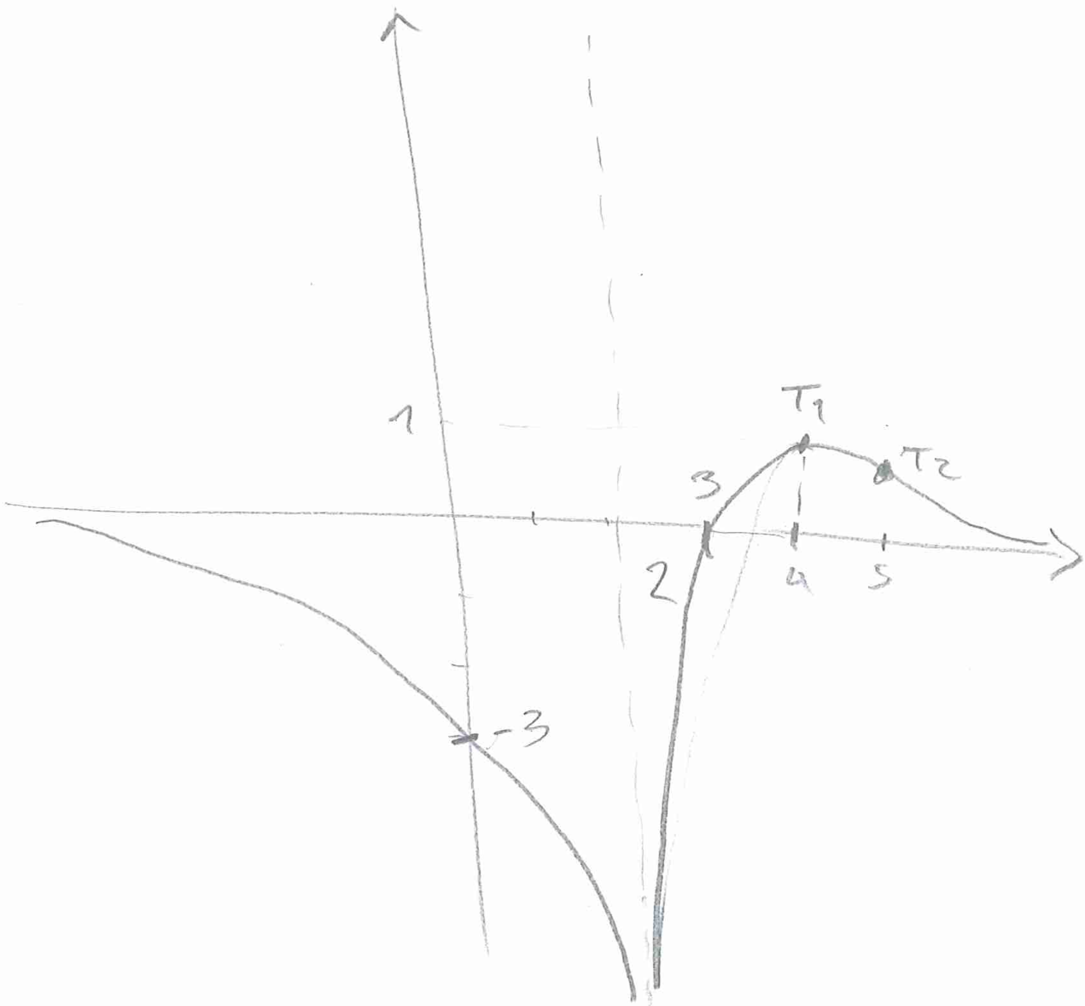
$$f''(x) = 4\,\frac{-(x-2)^{\cancel{2}} - 3\cancel{(x-2)}^{2}(4-x)}{(x-2)^{\cancel{6}\,4}} =$$

$$= 4 \cdot \frac{-x+2-12+3x}{(x-2)^4} = 4 \cdot \frac{2x-10}{(x-2)^4} =$$

$$= \frac{8(x-5)}{(x-2)^4}$$

$$\left.\begin{array}{l} x<5:\ f''(x)<0 \Rightarrow \text{konkavno} \\ x>5:\ f''(x)>0 \Rightarrow \text{konveksno} \end{array}\right\} T_2\left(5, \tfrac{8}{9}\right) \text{ je prevoj}$$

$f(0) = -3$



$Z_f = (-\infty, 1]$ 5

(3) $f(-1) = a + b\, tg\left(-\frac{\pi}{4}\right) = a - b$, $f(1) = a + b\, tg\frac{\pi}{4} = a + b$

$$\left.\begin{array}{l} \lim\limits_{x \uparrow -1} f(x) = -1 = a - b \\ \\ \lim\limits_{x \downarrow 1} f(x) = 1 = a + b \end{array}\right\} \Rightarrow \boxed{a = 0\,,\ b = 1}$$

(4) 10

a) $$\int_0^1 x^2 e^{-x}\,dx = -e^{-x}x^2 \Big|_0^1 + 2\int_0^1 x e^{-x}\,dx =$$

PER PARTES

$u = x^2$, $dv = e^{-x}dx$

$du = 2x\,dx$, $v = -e^{-x}$

$$= -e^{-1} + 2\left(-e^{-x}x \Big|_0^1 + \int_0^1 e^{-x}\,dx\right) =$$

PER PARTES

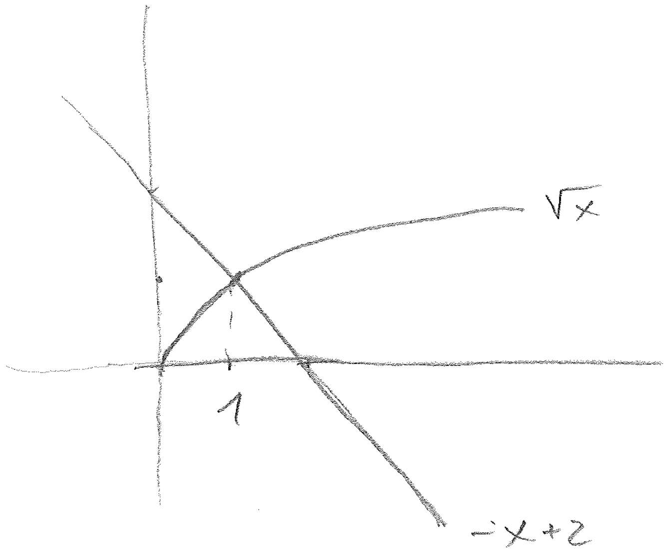
$u = x$, $dv = e^{-x}dx$

$du = dx$, $v = -e^{-x}dx$

$$= -e^{-1} + 2\left(-e^{-1} + e^{-x}\Big|_0^1\right) =$$

$$= -e^{-1} - 2e^{-1} - 2e^{-1} + 2 = \underline{\underline{2 - 5e^{-1}}}$$

b) (15)



$$\sqrt{x} = -x+2 \quad \Big|^2$$

$$x = x^2 - 4x + 4$$

$$x^2 - 5x + 4 = 0 \qquad (x-1)(x-4) = 0$$

$$x_1 = 1 \quad , \quad \underline{x_2 = 4} \Longrightarrow -x+2 < 0 \quad *$$

$$S = \int_0^1 \sqrt{x}\,dx + \int_1^2 (2-x)\,dx =$$

$$= \frac{2x^{3/2}}{3}\Big|_0^1 + \left(2x - \frac{x^2}{2}\right)\Big|_1^2 =$$

$$= \frac{2}{3} + 4 - 2 - \left(2 - \frac{1}{2}\right) = \frac{2}{3} + \frac{1}{2} = \underline{\underline{\frac{7}{6}}}$$