1. Prove that
$$2^{n-2}n(n-1) = \sum_{k=0}^{n} k(k-1)\binom{n}{k}.$$

2. Prove that for $k \leq n$ it holds
$$\binom{k}{k} + \binom{k+1}{k} + \cdots + \binom{n}{k} = \binom{n+1}{k+1}.$$

3. Prove that
$$\binom{n+m}{k} = \sum_{i} \binom{n}{i}\binom{m}{k-i}.$$

4. Prove that
$$B(n+1) = \sum_{i=0}^{n} \binom{n}{i} B(i).$$

5. Prove that $B(n) \leq n!$.

6. Prove that $n! \leq S(2n, n) \leq (2n)!$.

7. Let $F(n, k)$ denote the number of sequences $A_1, \ldots, A_n$, where $1 \leq A_i \leq k$, $A_i \in \mathbb{N}$, for all $i \in [n]$, and the following properties hold for the sequence.

   - For every $j \in [k]$ there exists $i \in [n]$ such that $A_i = j$.
   - The first time j appears in the sequence $A_1, \ldots, A_n$ is before the first appearance of $j + 1$.

   Prove that $F(n, k) = S(n, k)$.