1. Determine the number of ways in which we can place non-attacking rooks on a $n \times n$ triangular board.

2. Determine the number of partitions of n with $\lambda_1 - \lambda_2 \geq 3$.

3. Determine the number of partitions of n with $\lambda_1 - \lambda_2 = 3$.

4. Let $\lambda = (\lambda_1, \lambda_2, \ldots)$ be a partition. Prove the following:

   (a) $\displaystyle\sum_{i \geq 1} (i-1)\lambda_i = \sum_{i \geq 1} \binom{\lambda_i'}{2}$,

   (b) $\displaystyle\sum_{i \geq 1} \left\lfloor \frac{\lambda_{2i-1}}{2} \right\rfloor = \sum_{i \geq 1} \left\lceil \frac{\lambda_{2i}'}{2} \right\rceil$.

5. A standard Young tableau of shape λ, where λ is a partition of n, is a bijection $T\colon [\lambda] \to [n]$ for which it holds $T(i,j) < T(i,j')$ for $j < j'$, and $T(i,j) < T(i',j)$ for $i < i'$. The number of SYT of shape λ is f^λ. Prove that $f^{n,n} = C_n$.