1. Using egfs, solve the following recurrence equation $a_{n+1} = 2a_n + n$, $a_0 = 1$.
2. Let $A \subseteq \mathbb{N}$. Let a_n be the number of permutations of $[n]$ with lengths of cycles only from the set A. Determine an egf of $(a_n)_n$.
3. Determine the number of permutations in S_n which have an even number of cycles and each cycle is of odd length.
4. Let $H(x) = e^{F(x)}$ and $F(x) = \sum a_n \frac{x^n}{n!}$, $H(x) = \sum b_n \frac{x^n}{n!}$. Prove that

$$nb_n = \sum_k \binom{n}{k} k a_k b_{n-k}.$$