... Spuščena metoda: iščemo $\vartheta(t)$, $\varphi(t)$ ob pogojih

ob $t=0$: $\dot\varphi(0)=0$ & $\dot\vartheta(0)=0$

1. rešitev ✓

torej

$$0=\dot\varphi=\frac{b-a\cos\vartheta_1}{\sin^2\vartheta_1} \Rightarrow \boxed{b=au_1} \quad \mathrm{I}$$

ϑ_1 najvišja točka

$\vartheta(0)=\vartheta_1$

najvišji u

iz energije; $\dot\vartheta=0$, $\dot\varphi=0$

$$\tilde E=\tfrac{1}{2}J_1\dot\vartheta^2+\tilde V(\vartheta) \Rightarrow \tilde E = mgl\cos\vartheta_1$$

$\dot\vartheta^2 = 0$

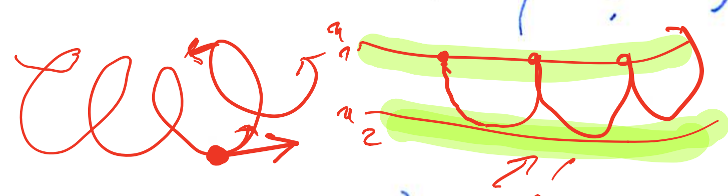
$$\tilde E - mgl\,u_1=0 \qquad \Big|\,\frac{2}{J_1}$$

$$\frac{2\tilde E}{J_1}-\frac{2mgl}{J_1}u_1=0$$

$$\boxed{\alpha-\beta u_1=0} \quad \mathrm{II}$$

$a, b, \alpha, \beta, \tilde E$

Iščemo še 2. rešitev $u_2=\cos\vartheta_2$ (spodnja meja za u)



$$f(u)=(1-u^2)(\underbrace{\alpha}_{\boxed{\beta u_1}\ \mathrm{II}}-\beta u)-(\underbrace{b}_{\boxed{au_1}\ \mathrm{I}}-au)^2=$$

$$=\beta(1-u^2)(u_1-u)-a^2(u_1-u)^2= \quad \left(\text{to je še točno}\right)$$

$u\to u_2$: $\quad = (u_1-u_2)\left(\beta(1-u_2^2)-a^2(u_1-u_2)\right)=0$

če $u_1\neq u$: $\quad \boxed{\beta(1-u_2^2)=a^2(u_1-u_2)}$

A B C

$$\beta u_2^2 - a^2 u_2 + 1\underbrace{(a^2 u_1 - \beta)}_{ab} = 0$$

že točno

$$u_2 = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A} = \frac{a^2 \pm \sqrt{a^4 - 4\beta(ab-\beta)}}{2\beta} = \dots$$

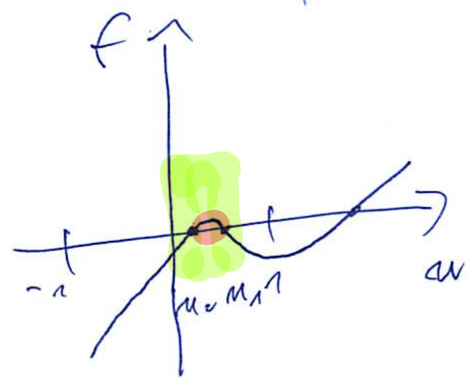
vzamemo rešitev $u_2 < u_1 = \frac{b}{a}$ ....

∷ Dodatna poenostavitev je hitra vrtavka:

D. N. vaje.

$$x = u_1 - u_2 = \frac{\beta}{a^2}(1 - u_2^2) > 0$$

če je $x \to 0$ vemo od prej, da velja $\frac{a^2}{\beta} > 2$, zelo hitra pa velja $\frac{a^2}{\beta} \gg 1$



torej

$$x = \underbrace{\frac{\beta}{a^2}}_{\ll 1}\underbrace{(1 - u_2^2)}_{<1} \ll 1 \qquad x \propto \frac{1}{\omega_3^2}$$

Če to velja, velja tudi $\dot u^2 \sim \dots u^2$ → nihanje

$$f(u) = (u_1 - u)\Big(\underbrace{\beta(1-u^2)}_{\text{malo}} - a^2\underbrace{(u_1 - u)}_{\text{veliko}}\Big) \approx$$

$$\approx a^2(u_1 - u)(u - u_2) \quad \text{sicer ne poznamo.}$$

[illegible]

vaje ali DN

$$\int \frac{du}{\sqrt{f(u)}} \Rightarrow \boxed{u = \frac{u_1 + u_2}{2} + \frac{x_1}{2}\cos at}$$

$p_\varphi = J_1 a$

Nutacija ~ precesija ni konstantna

$\vartheta(t)$

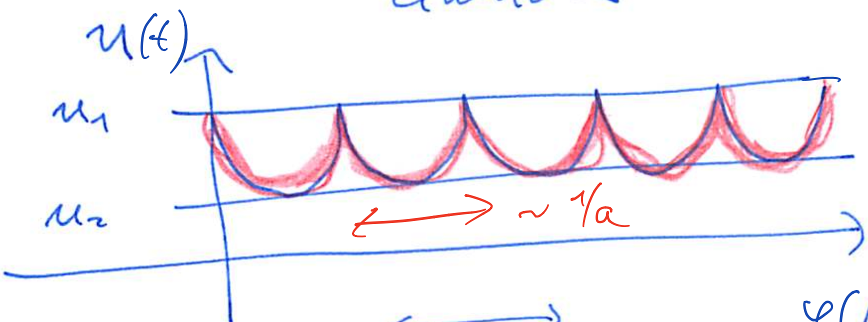
$$\Rightarrow \dot\varphi = \frac{b - au}{\sin^2\vartheta} = \frac{a(u_1 - u)}{\sin^2\vartheta} = \frac{a}{\sin^2\vartheta}\frac{x_1}{2}(1 - \cos at)$$

Precesija (povprečje):

$$\omega_p = \overline{\dot\varphi} = \frac{a x_1}{2\sin^2\vartheta} = \frac{\beta}{2a} = \frac{mgl}{J_3\omega_3} = \frac{M}{L_3}$$

kot v osnovni fiziki

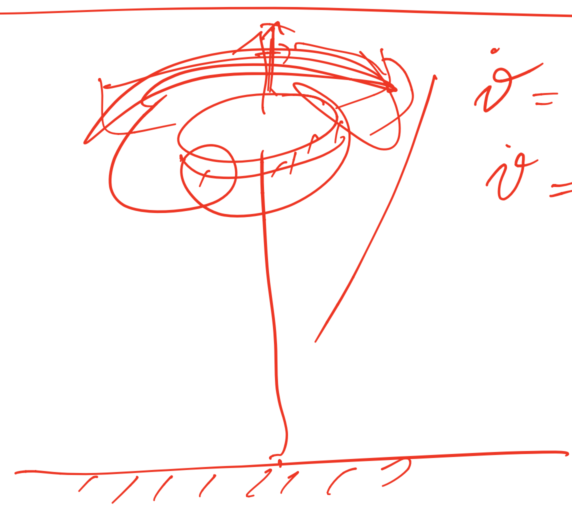
cikloida



$\} \propto \frac{1}{\omega_3^2} \to 0$ „hitra nutacija"

frekvenca $\propto a \to \infty$

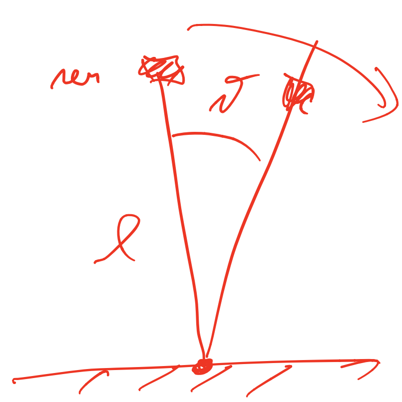
„hitrost" precesije $\sim \omega_\varphi \propto \frac{1}{\omega_3} \to 0$



$\vartheta = 0$

$\dot\vartheta = \delta \neq 0$

$T < \ldots$

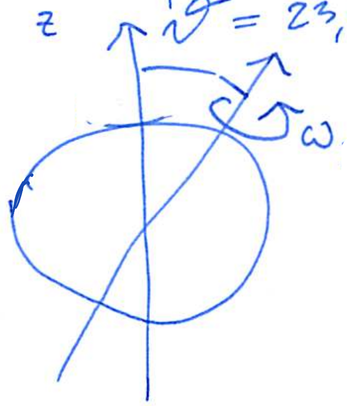


$$\ddot\vartheta - \omega^2\vartheta = 0$$

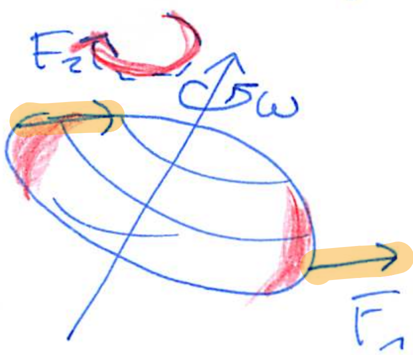
$$\vartheta \sim e^{\pm\omega t}$$

Primeri vrtavke – Zemlja

• Precesija zemljine osi



Os z je definirana glede na ekliptiko. Zaradi ω_z Zemlja ni okrogla, ampak geoid (~ elipsoid).



navor Sonca „hoče Zemljo poravnati“, zato precesira nazaj.

$F_1 > F_2$ (malo)

Sonce deluje z navorom, rezultat je precesija osi, ki se vrti nazaj glede na $\vec{\omega}$; „retrogradno“.

1 obrat = 25 700 let.

•• $\vartheta \sim 22{,}1^\circ \leftrightarrow 24{,}5^\circ$ zaradi drugih planetov; perioda je 41 000 let

∴ drugi planeti spreminjajo ekscentričnost ε. perioda je 105 000 let. Milankovićevi cikli