Hamilton-Jacobijeva enačba

neobvezno.

Spomnimo se akcije,

$$S = \int_{t_1}^{t_2} L(q, \dot{q}, t)\,dt,$$

$q(t)$ ✓

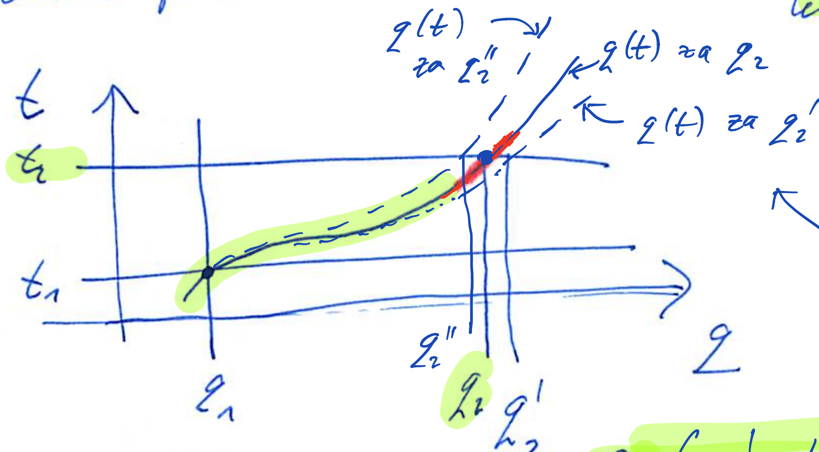
kjer je začetna in končna lega določena,

$q(t_1) = q_1$ in $q(t_2) = q_2$ in $q(t)$ zadošča enačbi gibanja ≡ „klasična pot".

Zamislimo si sedaj akcijo kot funkcijo klasične poti, tj. kjer je q_1 fiksirano in S gledamo kot funkcijo končne lege, pri čemer je $q(t)$ že klasična rešitev,

$$S = S(q_2, t_2)$$

Sedaj gremo s q_2' malo naprej ali malo nazaj od prave končne točke, vendar vzdolž prave poti pri fiksnem času t_2: klasična pot iz E-L. enačbe za dani q_2



za $\forall\, q_2$ je $q(t)$ različen

in analiziramo $S(q_2', t_2)$ kot funkcijo q_2

fiksiran

Od njej že znamo variirati S,

$$\delta S = \int_{t_1}^{t_2} \Big[\underbrace{\frac{\partial L}{\partial q} - \frac{d}{dt}\Big(\frac{\partial L}{\partial \dot q}\Big)}_{0} \Big] \delta q \, dt + \frac{\partial L}{\partial \dot q}\, \delta q(t) \Big|_{t_1}^{t_2} .$$

Vzdolž klasične poti $q(t)$ je prvi člen $\equiv 0$, zato ostane

$$\delta S = \frac{\partial L}{\partial \dot q}\Big|_{t_2} \delta q(t_2) - \frac{\partial L}{\partial \dot q}\, \underbrace{\delta q(t_1)}_{=0} =$$

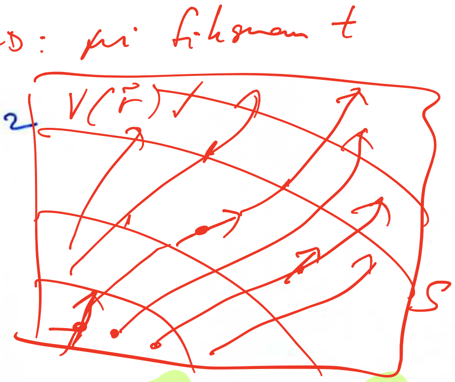
variiramo pri fiksnem t_2.

$q(t)$

$$= \frac{\partial L}{\partial \dot q}(t_2)\, \underbrace{\delta q(t_2)}_{q_2' - q_2}$$

$$\delta q \to 0 \longrightarrow \quad \frac{\partial S(q_2')}{\partial q_2'} = \frac{\partial L}{\partial \dot q}\Big|_{t=t_2} = p\Big|_{t=t_2} = p_2$$

2D: pri fiksnem t



Lahko zapišemo tudi

$$\frac{\partial S(q)}{\partial q} = \boxed{\frac{\partial S(q)}{\partial q} = p}$$

pri fiksnem $t = t_2$

, če namesto q_2' pišemo q, p_2 → p.

3 Dim: $\nabla S(\vec r) = \vec p$ $\quad \vec p(\vec r) = ?$

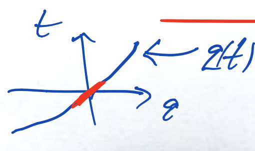
Poglejmo sedaj še

$$\frac{dS(q,t)}{dt} :$$

$$\boxed{\frac{dS}{dt_2}} = \frac{\partial S}{\partial t}\Big|_{t=t_2} + \frac{\partial S}{\partial q}\dot q\Big|_{t=t_2} = \frac{\partial S}{\partial t_2} + p\,\dot q\Big|_{t=t_2}$$

t $\quad t$ $\quad t$

$$dS = \frac{\partial S}{\partial t} dt + \frac{\partial S}{\partial q} \dot q \, dt$$



Po drugi strani velja itak

$$\frac{d}{dx}\int_0^x f(x')dx' = f(x)$$

$$\frac{dS}{dt_2} = L$$ (odvod po zgornji meji) pri poti $q(t)$

zato $$\boxed{L = \frac{\partial S}{\partial t_2} + p\dot{q}\Big|_{t_2}}$$ oz.)

$$\frac{\partial S}{\partial t_2} = -p\dot{q}\Big|_{t_2} + L(q_2, \dot{q}_2, t_2) =$$

$$= -(p\dot{q} - L) = -H(q_2, p_2, t_2).$$

Rezultat ($t_2 \to t$):

$$\boxed{H + \frac{\partial S}{\partial t} = 0 \quad \text{in} \quad \frac{\partial S}{\partial q}\Big|_{q(t)} = p}$$

gibalna količina odvisna od skalarnega polja tudi v 3D, $\vec{p} = \nabla S$.

$$S = \int_0^t L\,dt'$$ $q(t')$

$$H = H(q, p, t) = H\left(q, \frac{\partial S}{\partial q}, t\right)$$

H.-J. enačba: (a) diferencialna 1. reda

(b) samo funkcija od q

$S(q,t)$; $p = \frac{\partial S}{\partial q}$; $H\left(q, \frac{\partial S}{\partial q}, t\right) + \frac{\partial S}{\partial t} = 0$ q, t

To je poseben primer kanonične transformacije, ko je $K = 0$ in

$$F_2(q, P) \to F_1 = S$$

$$p = \frac{\partial S}{\partial q}$$ $\nabla S = \vec{v}(\vec{r})$

gibanje delca tore opiše nekakšno uvlovanje (polje).

$S(\vec{r})$, $m\dot{\vec{r}}$

$$K = 0 = H + \frac{\partial S}{\partial t}$$

če $$H = \frac{p^2}{2m} + V(q)$$

$$\Rightarrow \boxed{-\frac{\partial S}{\partial t} = \frac{1}{2m}\left(\frac{\partial S}{\partial q}\right)^2 + V(q)}$$

$$\frac{1}{2m}(\nabla S)^2 + V(\vec{r}) = -\frac{\partial S}{\partial t}$$

podoben izraz dobimo v QM za fazo, $\psi = R\,e^{iS}$

če $H \neq H(t)$ in zato

$H\left(q, \frac{\partial S}{\partial q}\right) = E$, je S „Hamiltonova glavna funkcija".

„principal"

{polje ↔ delec}

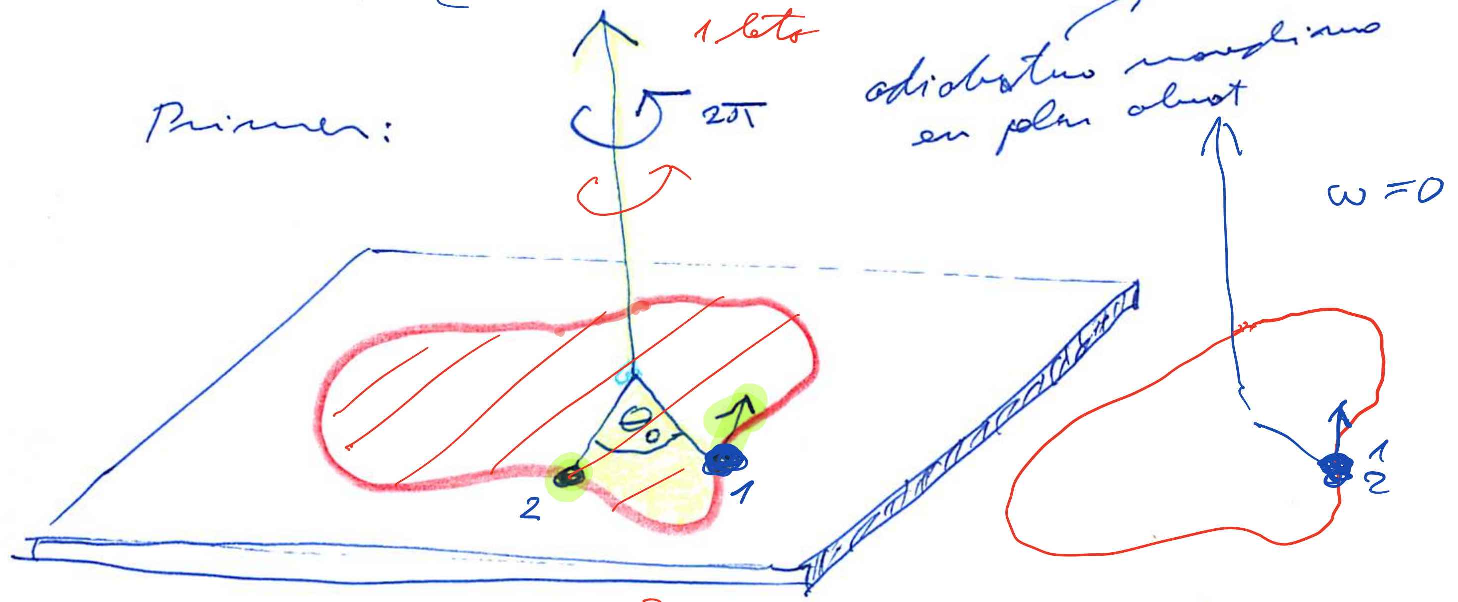
Hannayev kot (1984)

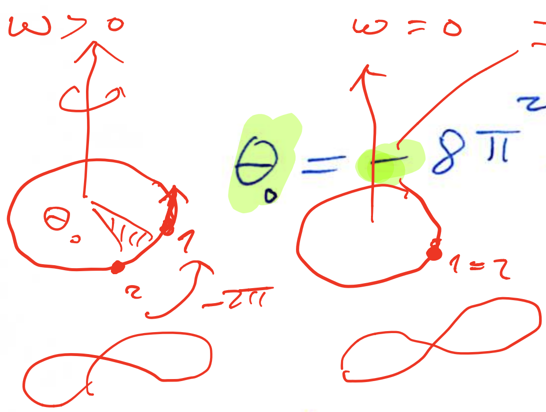
Akcija: $I = \frac{1}{2\pi}\oint p\,dq$

kot θ: $\frac{1}{\tau} \gtrsim \omega > 0$

(I, θ) formalizem

Primer:





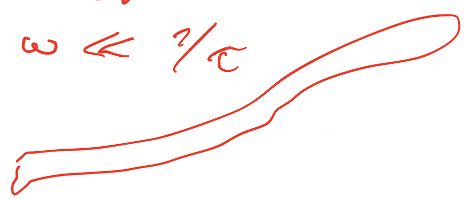
$$\Theta_0 = -8\pi^2 \frac{S}{\ell^2}$$

$\pm(\omega)$

ploščina zanke

obseg

adiabatno $\omega \ll 1/\tau$



Za krog, $S = \pi R^2$, $\ell = 2\pi R \Rightarrow \Theta_0 = -2\pi$.

Jules Verne,
V 80 dneh okoli sveta
(mi: ~~povsem~~ isto...)

— K O N E C —


AROUND the WORLD IN 80 DAYS
By JULES VERNE
IN THE YEAR 1872, THERE WERE NO GIGANTIC LINERS THAT CROSSED THE ATLANTIC OCEAN IN FOUR DAYS OR LESS; NOR WERE THERE ANY AIRPLANES THAT COULD FLY AT SPEEDS OF SIX HUNDRED MILES AN HOUR. IN FACT, THERE WERE NO AIRPLANES AT ALL. UNDER THE MOST FAVORABLE CONDITIONS, IT REQUIRED A MINIMUM OF THREE MONTHS OF MOST UNCOMFORTABLE TRAVELING TO MAKE A COMPLETE TOUR OF THE WORLD.
YET, HERE WE BEGIN THE TALE OF A MAN WHO WAGERED 20,000 ENGLISH POUNDS STERLING, THAT HE COULD MAKE THE TRIP
AROUND THE WORLD IN 80 DAYS.
Phileas Fogg
Passepartout
His faithful servant
Aouda
An Indian Princess
Detective Fix
Captain Speedy
Sir Francis
Illustrated by H.C. Kiefer

79