Kanonične transformacije

neobvezno

Recimo, da zamenjamo spremenljivke,

$q_i \to Q_i(\underline{q}, \underline{p}, t)$ in

$p_i \to P_i(\underline{q}, \underline{p}, t)$.

Transformacija med koordinatami je kanonična (def.), če ohranja Poissonove oklepaje,

$$\sum_i \left(\frac{\partial f}{\partial Q_i}\frac{\partial g}{\partial P_i} - \frac{\partial f}{\partial P_i}\frac{\partial g}{\partial Q_i}\right)$$

(q, p)

$$\boxed{\{f, g\}_{QP} \overset{?}{=} \{f, g\}_{qp}} = \sum_i \left(\frac{\partial f}{\partial q_i}\frac{\partial g}{\partial p_i} - \frac{\partial f}{\partial p_i}\frac{\partial g}{\partial q_i}\right)$$

$f(\underline{q}(\underline{Q}, \underline{P}), \underline{p}(\underline{Q}, \underline{P}), t) \equiv \tilde{f}(\underline{Q}, \underline{P}, t)$ $\qquad f(\underline{q}, p, t)$.

Če je tako, potem so H. enačbe invariantne na transformacijo v smislu

$$\dot{Q}_i = \{Q_i, H\}_{qp} + \frac{\partial Q_i}{\partial t} = \{Q_i, H\}_{QP} + \frac{\partial Q_i}{\partial t} = \frac{\partial H}{\partial P_i} + \frac{\partial Q_i}{\partial t}$$

naj bo $\frac{\partial Q_i}{\partial t} = 0$

$$\dot{P}_i = \{P_i, H\}_{qp} + \frac{\partial P_i}{\partial t} = \ldots = -\frac{\partial H}{\partial Q_i} + \frac{\partial P_i}{\partial t}$$

!

Če $\frac{\partial Q_i}{\partial t} = 0$ in $\frac{\partial P_i}{\partial t} = 0$, so enačbe enake. 0?

$\dot{q} = \frac{\partial H}{\partial p}, \quad \dot{p} = -\frac{\partial H}{\partial q}$

Kako se poišče nove koordinate ??

Nekoliko del. kanoničnost s pogojem

$$\boxed{\{Q_i, P_j\} = \delta_{ij}},$$

!

ki je ekvivalenten prejšnji definiciji.

$(q,p) \to (Q,P)$

$Q_i(\underline{q}, \underline{p}, t)$

P ?

K ?

Primer

Recimo, da imamo poseben primer, ko velja

poljubno: $\boxed{Q_i = Q_i(\underline{q}, t)}$ in $q_i = q_i(\underline{Q}, t)$.

L zapišimo tako: $L(\underline{q}(\underline{Q}, t), \dot{\underline{q}}(\underline{Q}, t), t)$

$\boxed{P_i, K}$ Kako najdemo $\boxed{P_i}$ $\boxed{?}$

Vemo, da lahko L-u prištejemo popoln odvod,

$$\rightarrow \quad \tilde{L} = L - \frac{dF}{dt}.$$

iščemo moment iščemo Hamiltonovo f.

Želimo $(\underline{Q}, \underline{P})$, $K(\underline{Q}, \underline{P}, t)$ in da bo

$K = P\dot{Q} - \tilde{L}$

$H = p\dot{q} - L$

$$\boxed{\dot{Q}_i = \frac{\partial K}{\partial P_i}} \quad \text{in} \quad \boxed{\dot{P}_i = -\frac{\partial K}{\partial Q_i}},$$

torej

$$\tilde{L} = \sum_i \dot{Q}_i P_i - K = \sum_i \dot{q}_i p_i - H - \frac{dF}{dt}.$$

- Izberimo poseben primer, $\boxed{F = F_1(\underline{q}, \underline{Q}, t)}$ in

$$\frac{dF_1}{dt} = \sum_i \left(\frac{\partial F_1}{\partial q_i} \dot{q}_i + \frac{\partial F_1}{\partial Q_i} \dot{Q}_i \right) + \frac{\partial F_1}{\partial t}$$

in vstavimo v gornjo enačbo. Dobimo

zveze:

$$\boxed{P_i = -\frac{\partial F_1}{\partial Q_i}}, \quad \boxed{p_i = +\frac{\partial F_1}{\partial q_i}}, \quad \boxed{K = H + \frac{\partial F_1}{\partial t}}.$$

Tako smo „našli“ P_i. Opomba: P_i so izraženi s q_i in Q_i.

v informacijo: receptura $q, p, H \to Q, P, K$

V splošnem imamo 4 možnosti za F, in da se da vse določiti:

$$1)\ \boxed{F = F_1(q, Q, t):\ p_i = \frac{\partial F_1}{\partial q_i},\ P_i = -\frac{\partial F_1}{\partial Q_i},\ K = H + \frac{\partial F_1}{\partial t}}$$

$$2)\ F_2(q, P, t) = F + QP:\ p_i = \frac{\partial F_2}{\partial q_i},\ Q_i = \frac{\partial F_2}{\partial P_i},\ K = H + \frac{\partial F_2}{\partial t}$$

$$3)\ F_3(p, Q, t) = F - qp:\ q_i = -\frac{\partial F_3}{\partial p_i},\ P_i = -\frac{\partial F_3}{\partial Q_i},\ K = H + \frac{\partial F_3}{\partial t}$$

$$4)\ F_4(p, P, t) = F_2 - qp:\ q_i = -\frac{\partial F_4}{\partial p_i},\ Q_i = \frac{\partial F_4}{\partial P_i},\ K = H + \frac{\partial F_4}{\partial t}$$

Legendrove transformacije.