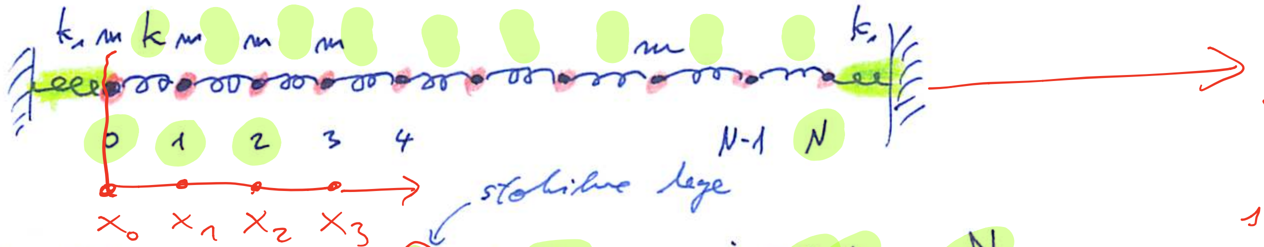
Dinamika zveznega sredstva

Poglejmo si najprej eno vajo: gibalne enačbe za verigo nihal. 1 dim.



$L = T - V$; $\quad q_i = \underbrace{i a}_{q_i^0} + u_i$; $\quad i = 0, 1, \dots N$ $\qquad \frac{\partial u(x,t)}{\partial x} \sim \frac{\Delta u}{\Delta x}$, $\Delta x \to 0$

$$L = \frac{1}{2}\sum_{i=0}^{N} m \dot{u}_i^2 - \frac{1}{2}\sum_{i=0}^{N-1} k(u_{i+1} - u_i)^2 \cancel{- \frac{1}{2} k_1 (u_0^2 + u_N^2)}.$$

„robni" pogoji npr.: (a) $k_1 \to 0$, veriga se lahko translatira, vendar končni k_1 poskrbi za definirane ravnovesne lege

(b) $k_1 \to \infty$, togo vpeti robni masi.

(c) linearne kombinacije u in $\dot{u}$ na robu itd.

Enačbe gibanja,

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{u}_i} - \frac{\partial L}{\partial u_i} = 0 :$$

$$m\ddot{u}_i + k(u_{i+1} - u_i)(-1) + k(u_i - u_{i-1}) = 0, \text{ če } i \neq 0, N$$

oz.

$$\boxed{m\ddot{u}_i = k(u_{i-1} - 2u_i + u_{i+1})}.$$

enako, ker $N \to \infty$; $a \to 0$

Če $N \to \infty$, lahko pri fiksni celotni dolžini X vse skupaj obravnavamo kot palico z vzdolžnim nihanjem,

$$a = \frac{X}{N}, \quad m = \frac{M_{palica}}{N+1}.$$

Spomnimo se receptur za numerično odvajanje:

$$\frac{df(x)}{dx} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$\boxed{\frac{d^2 f(x)}{dx^2} = \lim_{\Delta x \to 0} \frac{f'(x+\Delta x) - f'(x)}{\Delta x}} =$$

$$= \lim_{\Delta x \to 0} \frac{f(x+2\Delta x) - f(x+\Delta x) - f(x+\Delta x) + f(x)}{(\Delta x)^2} =$$

$$\cong \left[f(x+2\Delta x) - 2f(x+\Delta x) + f(x) \right] / (\Delta x)^2$$

Pri nas vzamemo $x = i a$ in $\in N$ $x \in R$

$u(x,t) = u_i(t)$; $(N \to \infty)$ 10^{20}

mora na robu sedaj „robni pogoji"!

Δx

$$\boxed{m \ddot{u}(x,t) = k a^2 \frac{\partial^2}{\partial x^2} u(x,t)}$$

$$\boxed{\frac{\partial^2 u}{\partial t^2} = u_{tt}}$$

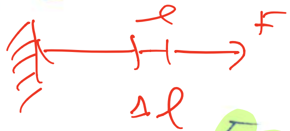
$x = ia \in R$

$$\boxed{\frac{\partial^2 u}{\partial x^2} = u_{xx}}$$

$$\boxed{u_{tt} = \underbrace{k \frac{a^2}{m}}_{c^2} u_{xx}}$$ običajna valovna enačba

c^2 hitrost valovanja

Hitrost valovanja povezali bomo z znanimi konstantami. Torej



$$\boxed{\frac{\Delta l}{l} = \frac{F}{S_0 E_Y}}$$

E_Y ~ Youngov modul (prožnostni)

S_0 ~ presek palice

$$\frac{u_i}{a} = \frac{k u_i}{S_0 E_Y} \Rightarrow k = \frac{S_0 E_Y}{a}$$

$$m = \rho S_0 a, \quad \boxed{\frac{k a^2}{m} = \frac{S_0 E_Y a^2}{\rho S_0 a a} = \frac{E_Y}{\rho} = c^2}$$

(znan rezultat)

Sedaj poiščimo L, da bo valovna enačba „enačba gibanja". Imamo že gostoto mase, E. Definiramo ~~gostoto~~ gostoto energije

ER

$L \rightarrow \ddot{u} = c^2 u''$

$u_{tt} = c^2 u_{xx}$

$u(x,t)$ ~~q_i~~

$u_i(t)$

$$\frac{T}{S_0 a} = \mathcal{T} = \frac{1}{2}\frac{m}{S_0 a} u_t^2 = \frac{1}{2}\rho u_t^2 \quad ; \quad u_t = \frac{\partial u}{\partial t} = \dot{u}(x,t)$$

$$\mathcal{V} = \frac{V}{S_0 a} = \frac{1}{2} E_Y u_x^2 .$$

Torej Lagrangeova gostota

$$\mathcal{L} = \frac{1}{2}\rho u_t^2 - \frac{1}{2} E_Y u_x^2 \quad \text{in}$$

$\left(\lim_{\substack{N\to\infty \\ a\to 0}}\right)$

$$\boxed{L = S_0 \int_0^X \mathcal{L}(u, u_x, u_t, x, t)\, dx} . \quad \left(= \int \mathcal{L}\, d^3 r\right)$$

novo!

L. funkcija.

To je za 1 dimenzija.

običajno v L. funkciji nastopajo

q_i, $\dot{q}_i$ in t : $L = L(\underline{q}, \underline{\dot{q}}, t)$.

Tukaj imamo zvezen indeks „i", v smislu

$$x_i = ia \xrightarrow{a \to 0} x \in \mathbb{R}$$

in

$$q_i(t) \leftrightarrow u_i(t) \leftrightarrow u(x_i, t) \leftrightarrow u(x,t).$$

V Lagrangeovi gostoti simetrično nastopata x in t, tudi odvoda:

$$\mathcal{L} = \mathcal{L}\left(u(x,t), \frac{\partial u}{\partial x}, \frac{\partial u}{\partial t}, x, t\right).$$

Iz tega bomo izpeljali enačbe gibanja $\longleftrightarrow$ valovno enačbo.