1. $(\vec{x} \times \vec{a}) \times \vec{a} = \vec{x} \times \vec{b}$

1.1. $\vec{a}, \vec{b}$ lin. odv. 10

1.1.1 $\vec{a} = 0$ 1 $\Rightarrow \vec{x} \times \vec{b} = 0$ 1 $\Rightarrow \vec{x} = \lambda \vec{b}$, za $\vec{b} \neq 0$ sicer $\vec{x} \in \mathbb{R}^3$ 2

1.1.2 $\vec{a} \neq 0$ 1 $\Rightarrow \vec{b} = \lambda \vec{a}$ 1 $\Rightarrow (\vec{x} \times \vec{a}) \times \vec{a} = \lambda (\vec{x} \times \vec{a})$

$\Rightarrow \vec{x} \times \vec{a} \perp \vec{x} \times \vec{a}$ 2

$\Rightarrow \vec{x} \times \vec{a} = 0$ 1

$\Rightarrow \vec{x} = \alpha \vec{a}$ 1

1.2. $\vec{a}, \vec{b}$ lin. neodv. 15

$\vec{x} = \alpha \vec{a} + \beta \vec{b} + \gamma (\vec{a} \times \vec{b})$ 2

$(\vec{x}\vec{a})\vec{a} - |\vec{a}|^2 \vec{x} = \vec{x} \times \vec{b}$ 2

$(\alpha |\vec{a}|^2 + \beta \vec{a}\vec{b})\vec{a} - |\vec{a}|^2 (\alpha \vec{a} + \beta \vec{b} + \gamma \vec{a} \times \vec{b})$

$= \alpha \vec{a} \times \vec{b} + \gamma ((\vec{a}\vec{b})\vec{b} - |\vec{b}|^2 \vec{a})$ 2

$\vec{a}$: $\beta \vec{a}\vec{b} = -\gamma |\vec{b}|^2$

$\vec{b}$: $-\beta |\vec{a}|^2 = \gamma (\vec{a}\vec{b})$ 2

$\vec{a} \times \vec{b}$: $-\gamma |\vec{a}|^2 = \alpha$

$\Rightarrow \gamma = -\beta \frac{\vec{a}\vec{b}}{|\vec{b}|^2} = \frac{\gamma (\vec{a}\vec{b})^2}{|\vec{b}|^2 |\vec{a}|^2} \Rightarrow \gamma \left(1 - \frac{(\vec{a}\vec{b})^2}{|\vec{a}|^2 |\vec{b}|^2}\right) = 0$

2

1.2.1 $\gamma = 0 \Rightarrow \alpha = 0 \Rightarrow \beta = 0 \Rightarrow \vec{x} = 0$ 2

1.2.2 $\gamma \neq 0 \Rightarrow (\vec{a}\vec{b})^2 = |\vec{a}|^2 |\vec{b}|^2 = |\vec{a}|^2 |\vec{b}|^2 \cos^2 \varphi$

$\Rightarrow \cos^2 \varphi = 1 \Rightarrow \varphi \in \{0, \pi\}$

$\Rightarrow \vec{a} \parallel \vec{b}$ ↯. 3

2. a) $\vec{a} = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$, $\vec{b} = \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}$, $\vec{c} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ so pravokotni

$\vec{a}, \vec{b} \in \Sigma$, $\vec{c} \perp \Sigma$ 2

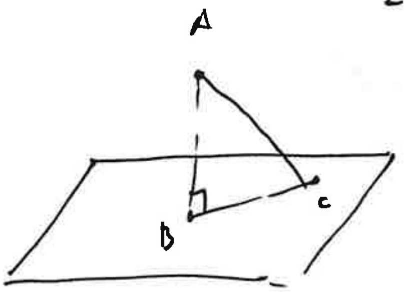
$P_{\mathcal{BB}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ 2 $\quad Q_{\mathcal{SB}} = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} \\ 0 & -\frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} \end{bmatrix}$ 3 je preh. matr.

$$P_{\mathcal{SS}} = Q_{\mathcal{SB}} P_{\mathcal{BB}} \cdot Q_{\mathcal{BS}} = \begin{bmatrix} 2/3 & -1/3 & -1/3 \\ -1/3 & 2/3 & -1/3 \\ -1/3 & -1/3 & 2/3 \end{bmatrix}$$ 5

$$Q_{\mathcal{BS}} = Q_{\mathcal{SB}}^{-1} = Q_{\mathcal{SB}}^{T}$$

Opomba: lahko rešimo tudi s projekcijami $\vec{e}_1$, $\vec{e}_2$ in $\vec{e}_3$ na Σ.

b) $$\vec{r}_B = P_{\mathcal{SS}} \vec{r}_A = \begin{bmatrix} 2/3 & -1/3 & -1/3 \\ -1/3 & 2/3 & -1/3 \\ -1/3 & -1/3 & 2/3 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2/3 \\ -1/3 \\ -1/3 \end{bmatrix}$$ 5

c) 

Da je ΔABC enakokrak, je B ~~trikotnik~~ vrh trikotnika. 3 Iskane točke C so $C \in \Sigma$, do je

$$|\vec{BC}| = |\vec{AB}| = |\vec{r}_B - \vec{r}_A| = \left| \begin{bmatrix} 4/3 \\ 4/3 \\ 4/3 \end{bmatrix} \right| =$$

$$\sqrt{3 \cdot \frac{16}{9}} = \frac{4}{\sqrt{3}}$$ 3

Krožnica na Σ s središčem v B in polmerom $\frac{4}{\sqrt{3}}$.

2

3. $\det(A-\lambda I) = \begin{vmatrix} 4-\lambda & 1 & 1 & 0 \\ -1 & 4-\lambda & 0 & -1 \\ 1 & 0 & 4-\lambda & 1 \\ 0 & -1 & -1 & 4-\lambda \end{vmatrix} = \dots = (4-\lambda)^4$ 4

$\lambda_{1,2,3,4} = 4$

$\ker(A-4I)$:

$$\begin{bmatrix} 0 & 1 & 1 & 0 \\ -1 & 0 & 0 & -1 \\ 1 & 0 & 0 & 1 \\ 0 & -1 & -1 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$ 2

lastna vekt. $v_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix}, \quad v_2 = \begin{bmatrix} 0 \\ 1 \\ -1 \\ 0 \end{bmatrix}$ 2 ← dve kletki

$(A-4I)^2 = 0$ 2 $\Rightarrow$ največja kletka je velikosti 2×2.

$$\Rightarrow \quad J = \left[\begin{array}{cc|cc} 4 & 1 & & \\ & 4 & & \\ \hline & & 4 & 1 \\ & & & 4 \end{array}\right]$$ 3

$$\log_2(J) = \left[\begin{array}{cc|cc} 2 & \frac{1}{4\ln 2} & & \\ & 2 & & \\ \hline & & 2 & \frac{1}{4\ln 2} \\ & & & 2 \end{array}\right]$$ 3

$f(x) = \log_2(x)$
$f(4) = \log_2(4) = 2$ 2
$f'(4) = \log_2{}'(4) = \frac{1}{4\ln 2}$

P: $\{v_1, v_2\}$ dop. do baze z $\{v_3, v_4\}$; $v_3 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \quad v_4 = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}$ 3

$(A-4I)v_3 = \begin{bmatrix} 0 \\ -2 \\ 2 \\ 0 \end{bmatrix} = -2v_2$ 2

$(A-4I)v_4 = \begin{bmatrix} 2 \\ 0 \\ 0 \\ -2 \end{bmatrix} = 2v_1$

$$\Rightarrow \quad P = \begin{bmatrix} 0 & 1 & 2 & 0 \\ -2 & 0 & 0 & 1 \\ 2 & 0 & 0 & 1 \\ 0 & 1 & -2 & 0 \end{bmatrix}$$ 2

4. $\det(A-\lambda I) = \begin{vmatrix} 2-\lambda & -1 & 1 \\ 2-i & -1+i-\lambda & 1 \\ 2-i & -2+i & 2-\lambda \end{vmatrix} = \dots = (1-\lambda)(i-\lambda)(2-\lambda)$

$\lambda_1 = 1, \quad \lambda_2 = i, \quad \lambda_3 = 2$ 4

lastni vektorji:

$\lambda_1 = 1 \Rightarrow v_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$

$\lambda_2 = i \Rightarrow v_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$ 6

$\lambda_3 = 2 \Rightarrow v_3 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

Ker A normalna in $\lambda_i \neq \lambda_j$ za $i \neq j$, so v_1, v_2, v_3 paroma pravokotni. 3

$$\left\langle \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \right\rangle = \left\langle \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} - \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \right\rangle =$$

$$= \left\langle \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \right\rangle = \left\| \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \right\|^2 = 0+1+1 = 2 \quad 3$$

$$\left\| \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} \right\| = \left\| \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \right\| = \sqrt{(1+1+0)+(0+1+1)} = 2 \quad 3$$

$$\left\| \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \right\| = \left\| \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} - \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \right\| = \sqrt{(1+1+1)+(0+1+1)} = \sqrt{5} \quad 3$$

$$\cos\varphi = \frac{\left\langle \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \right\rangle}{\left\| \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} \right\| \cdot \left\| \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \right\|} = \frac{2}{2\sqrt{5}} = \frac{1}{\sqrt{5}} \Rightarrow \varphi = \arccos\frac{1}{\sqrt{5}}$$ 3