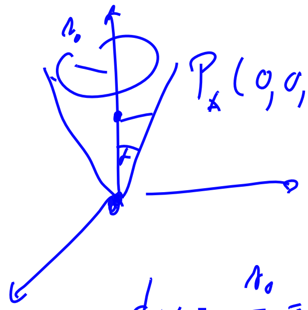


$P_*(0,0,\frac{3}{4}h)$

$\mathrm{tg}\,\alpha = \frac{r_0}{h} = \frac{r}{z}$

$r = \frac{r_0}{h} z$

$V = \frac{1}{3} r_0^2 h \pi$

Vaje 29. december 2020

1. Izračunaj vztrajnostni tenzor stožca:

   (a) s polom v vrhu stožca;

$$J_1 = J_2 = \frac{3}{5}m\left(\frac{1}{4}r_0^2 + \cancel{3}h^2\right), \qquad J_3 = \frac{3}{10}mr_0^2.$$

   (b) s polom v masnem središču;

   (c) v koordinatnem sistemu z eno osjo v smeri tvorilke stožca.

$$z_* = \frac{1}{\rho V}\int_{\mathcal{D}} z\rho\, dV = \frac{1}{V}\int_0^h dz \int_0^{2\pi} d\varphi \int_0^{\frac{r_0 z}{h}} r\,dr \cdot z = \frac{1}{V}2\pi\int_0^h z\,\frac{1}{2}\frac{r_0^2 z^2}{h^2}\,dz =$$

$$= \frac{1}{V}\pi\frac{r_0^2}{h^2}\cdot\frac{1}{4}h^4 = \frac{1}{4V}\pi r_0^2 h^2 = \underline{\underline{\frac{3}{4}h}}$$

$$\underline{J}(P_0) = \underline{J}(P_*) + m|P_0 - P_*|^2\underline{I} - m(P_0 - P_*)\otimes(P_0 - P_*)$$

$$\underline{J}(P_*) = \begin{bmatrix} J_1 & & \\ & J_2 & \\ & & J_3 \end{bmatrix} - m\frac{9}{16}h^2\begin{bmatrix} 1 & & \\ & 1 & \\ & & 1 \end{bmatrix} + m\frac{9}{16}h^2\,\vec{k}\otimes\vec{k}$$

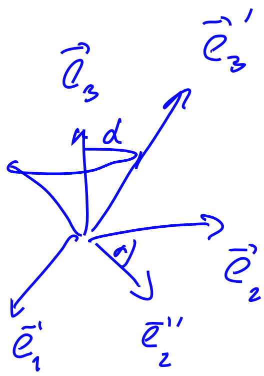
$$\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} J_1 & & \\ & J_2 & \\ & & J_3 \end{bmatrix} - m\frac{9}{16}h^2\begin{bmatrix} 1 & & \\ & 1 & \\ & & 0 \end{bmatrix}$$

$$J_1^* = J_2^* = \frac{3}{5}m\left(\frac{1}{4}r_0^2 + \cancel{3}h^2\right) - \frac{9}{16}mh^2 = \frac{3}{5}m\left(\frac{1}{4}r_0^2 + \frac{1}{16}h^2\right)$$

$$\left(\frac{3}{5} - \frac{9}{16}\right)mh^2 = \frac{48-45}{80}mh^2$$

$$J_3^* = J_3$$



$$\boxed{J'_{kl} = \alpha_{ki}\,\alpha_{lj}\,J_{ij}}$$

$$\vec{e}_k\,' = \alpha_{ki}\,\vec{e}_i$$

$$\vec{e}_1\,' = \vec{e}_1\,; \quad \vec{e}_2\,' = \cos\alpha\,\vec{e}_2 - \sin\alpha\,\vec{e}_3$$

$$\vec{e}_3{}' = \sin\alpha\,\vec{e}_2 + \cos\alpha\,\vec{e}_3$$

$$J'_{11} = \alpha_{1i}\alpha_{1j}J_{ij} = J_{11} \qquad \alpha_{11} = 1, \quad \alpha_{1j} = 0 \quad j = 2,3$$

$$J'_{12} = \alpha_{1i}\alpha_{2j}J_{ij} = \alpha_{11}\overset{0}{\overset{\shortparallel}{\alpha_{21}}}J_{11} = 0$$

$$J'_{13} = \alpha_{1i}\alpha_{3j}J_{ij} = \alpha_{11}\overset{0}{\overset{\shortparallel}{\alpha_{31}}}J_{11} = 0$$

$$J'_{22} = \alpha_{2i}\alpha_{2j}J_{ij} = \sum_{i=1}^{3}\alpha_{2i}^2 \underline{J_{ii}} = \alpha_{22}^2 J_{22} + \alpha_{23}^2 J_{33} = \cos^2\alpha\, J_{22} + \sin^2\alpha\, J_{33}$$

$$J'_{23} = \alpha_{2i}\alpha_{3j}J_{ij} = \sum_{i=1}^{3}\alpha_{2i}\alpha_{3i}J_{ii} = \alpha_{22}\alpha_{32}J_{22} + \alpha_{23}\alpha_{33}J_{33} =$$

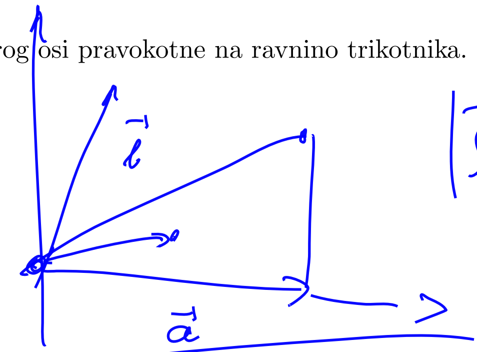
$$= \cos\alpha\sin\alpha\,(J_{22} - J_{33})$$

$$J'_{33} = \alpha_{3i}\alpha_{3j}J_{ij} = \sum_{i=1}^{3}\alpha_{3i}\alpha_{3i}J_{ii} = \alpha_{32}^2 J_{22} + \alpha_{33}^2 J_{33}$$

$$= \sin^2\alpha\, J_{22} + \cos^2\alpha\, J_{33} = \dots$$

$$\begin{bmatrix} J'_{11} & 0 & 0 \\ 0 & J'_{22} & J'_{11} \\ 0 & J'_{21} & J'_{33} \end{bmatrix}$$

2. Izračunaj vztrajnostni moment trikotne plošče okrog osi pravokotne na ravnino trikotnika.



$$\left|\xi \cdot \frac{\vec a}{|\vec a|}\right|^2$$

$$J_{33} = \int_A |\vec\xi|^2 \rho \, dA$$

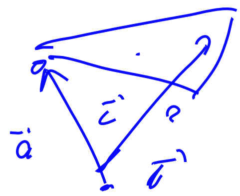
$$\vec\xi = \xi_2 \vec a + \xi_3 \vec b$$

$$dA = \left|\frac{\partial\vec\xi}{\partial\xi_2}\times\frac{\partial\vec\xi}{\partial\xi_3}\right| d\xi_2\, d\xi_3 = |\vec a\times\vec b|\, d\xi_2\, d\xi_3$$

$$|A| = \int_A dA = \int_0^1 d\xi_2 \int_0^{1-\xi_2} d\xi_3\, |\vec a\times\vec b| =$$

$$= |\vec a\times\vec b| \int_0^1 (1-\xi_2)\, d\xi_2 = |\vec a\times\vec b| \left(1-\frac12\right)$$

$$|A| = \frac12 |\vec a\times\vec b|$$



$$\vec r = \xi_1\vec a + \xi_2\vec b + \xi_3\vec c$$

$$\xi_1+\xi_2+\xi_3 = 1$$

$$J_{33} = \rho |\vec a\times\vec b| \int_0^1 d\xi_2 \int_0^{1-\xi_2} d\xi_3\, |\vec\xi|^2 = 2m \int_0^1 d\xi_2 \int_0^{1-\xi_2} d\xi_3 \left(|\vec a|^2\xi_2^2 + 2\vec a\cdot\vec b\,\xi_2\xi_3 + |\vec b|^2\xi_3^2\right)$$

$$= 2m\left[|\vec a|^2 \int_0^1 \xi_2^2(1-\xi_2)\, d\xi_2 + 2\vec a\cdot\vec b \int_0^1 \xi_2 \frac12 (1-\xi_2)^2 d\xi_2 + |\vec b|^2 \int_0^1 \frac13 (1-\xi_2)^3 d\xi_2\right]$$

$1-\xi_2 = u$

$\int_0^1 u^3 du$

$$= 2m\left[|\vec a|^2\left(\frac13-\frac14\right) + \vec a\cdot\vec b\left(\frac12 - 2\,\frac13 + \frac14\right) + |\vec b|^2 \frac13\cdot\frac14\right] =$$

$$= 2m\left[|\vec a|^2 \frac1{12} + \vec a\cdot\vec b\,\frac{6-8+3}{12} + \frac1{12}|\vec b|^2\right] = \frac16 m\left(|\vec a|^2 + \vec a\cdot\vec b + |\vec b|^2\right)$$

3. Ugotovi kdaj ima sistem sil $\mathcal{F}$ skupno prijemališče.

$$\mathcal{F} = \{(P_1, \vec F_1), \dots, (P_n, \vec F_n)\} \qquad \vec R(\mathcal F) = \sum_{i=1}^{n} \vec F_i$$

$$\vec N(\mathcal F, O) = \sum_{i=1}^{n} (P_i - O)\times \vec F_i$$

Ima skupno prijemališče, če obstaja točka P_0 tako, da je $\vec N(\mathcal F, P_0) = \vec 0$

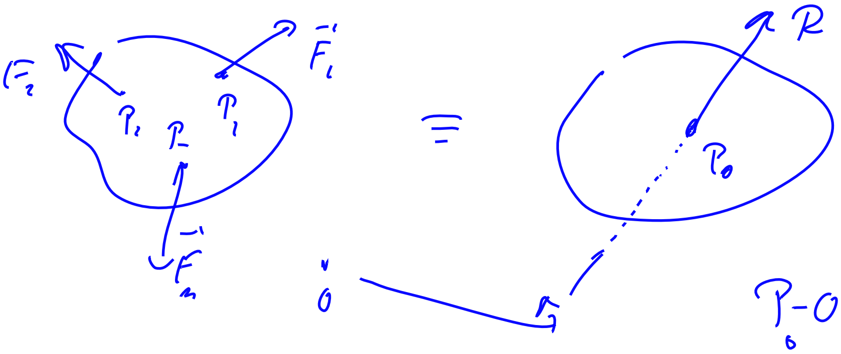
$$\vec 0 = \vec N(\mathcal F, P_0) = \vec N(\mathcal F, O) + (O - P_0)\times \vec R(\mathcal F) \qquad /\times (O-P_0)$$

$$\vec 0 = \vec N \times (O-P_0) + ((O-P_0)\times\vec R)\times(O-P_0)$$

$$/\times \vec R$$

$$\vec 0 = \vec N\times\vec R + ((O-P_0)\times \vec R)\times \vec R$$

$$\vec 0 = \vec N\times\vec R + \underbrace{((O-P_0)\cdot\vec R)\vec R} - |\vec R|^2 (O-P_0)$$



$$\vec 0 = \vec N\times\vec R - |\vec R|^2(O-P_0)$$

$$O-P_0 = \frac{\vec N\times\vec R}{|\vec R|^2}$$

$$P_0 - O = \frac{\vec R\times\vec N}{|\vec R|^2}$$

$$\vec N + (O-P_0)\times\vec R =$$

$$\vec N + \frac{1}{|\vec R|^2}(\vec N\times\vec R)\times\vec R = \vec N + \frac{1}{|\vec R|^2}\left((\vec N\cdot\vec R)\vec R - |\vec R|^2\vec N\right) = \frac{1}{|\vec R|^2}(\vec N\cdot\vec R)\vec R$$

P_0 je skupno prijemališče sil, če je $\boxed{\vec N\cdot\vec R = 0}$.

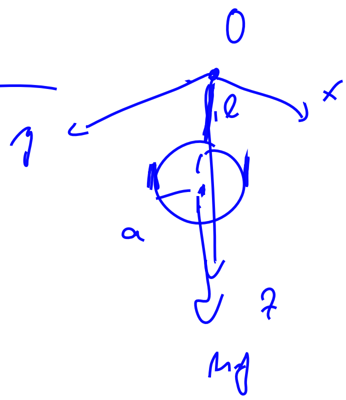
$$\vec N(\mathcal F, O)\cdot\vec R(\mathcal F) = 0$$

$I(\mathcal F) = \vec N(\mathcal F, O)\cdot \vec R(\mathcal F)$ invarianta sistema sil (je neodvisna od pola)

$$\vec N(\mathcal F, c) = \vec N(\mathcal F, \hat o) + \underline{(\hat o - c) \times \vec R(\mathcal F)}$$

$$\boxed{\vec R \neq \vec 0 \quad \text{i–} \quad I(\mathcal F) = 0}$$

4. Določi silo vrvice na kroglo v vogalu med dvema pravokotnima stenama.



$$\vec F_1 = F_1 \vec j \, ; \qquad \vec F_2 = F_2 \vec i \, ;$$

$$\vec F = F \frac{O - P_0}{|O - P_0|} \qquad \vec F_g = mg \vec k$$

$$\underline{\vec F_1 + \vec F_2 + \vec F + \vec F_g = \vec 0} \, ; \qquad \vec N(\mathcal F, P_0) = \vec 0$$

trivialno izpolnj–

$$P_0(a, a, h) \qquad |\vec{OP_0}|^2 = \underline{a^2 + a^2 + h^2 = (l+a)^2}$$

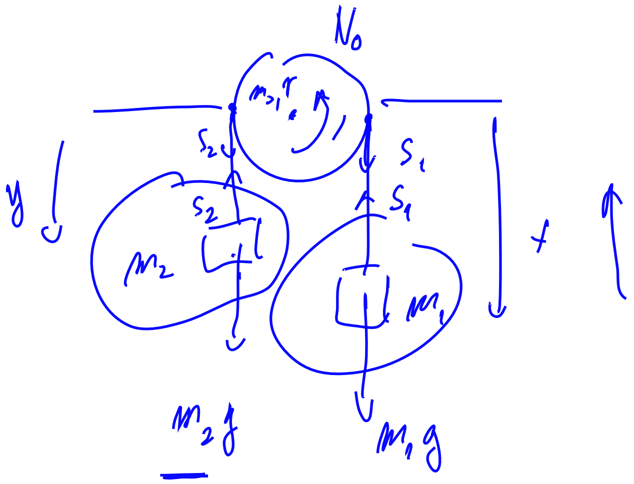
$$F_1 \vec j + F_2 \vec i + F \frac{1}{l+a}(-a\vec i - a\vec j - h\vec k) + mg\vec k = \vec 0$$

$$\left.\begin{aligned} F_2 - \frac{a}{l+a} F &= 0 \\ F_1 - \frac{a}{l+a} F &= 0 \end{aligned}\right\} \Rightarrow \quad F_1 = F_2 = \frac{a}{l+a} \cdot mg \frac{(l+a)}{h} = \frac{a}{h} mg$$

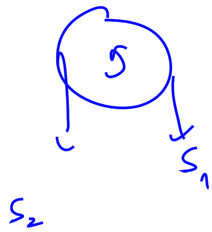
$$-\frac{h}{l+a} F + mg = 0 \quad \Rightarrow \quad F = \frac{mg(l+a)}{h}$$

$$2a^2 + h^2 = l^2 + 2al + a^2 \Rightarrow \underline{h = \sqrt{l^2 + 2al - a^2}}$$

5. Škripec z motorjem.



$$\begin{cases} m_1 \ddot{x} = m_1 g - S_1 \\ m_2 \ddot{y} = m_2 g - S_2 \\ J \ddot{\varphi} = r S_2 - r S_1 + N_0 \end{cases} ; \quad J = \frac{1}{2} m_3 r^2$$



$\underline{x, y, \varphi, S_1, S_2}$

$$\dot{x} = -\dot{y}$$

$$l = x + y + \pi r \Rightarrow \dot{x} + \dot{y} = 0$$

$$\boxed{\dot{y} = r \dot{\varphi}}$$

$$\frac{S_1}{m_1} = g - \ddot{x}$$

$$m_2 = 2 m_1, \quad m_3 = m_1$$

$$\ddot{x} = g - \frac{S_1}{m_1}$$

$$-\ddot{x} = g - \frac{S_2}{2 m_1} \Rightarrow \frac{S_2}{m_1} = 2 (g + \ddot{x})$$

$$-\frac{1}{2} m_1 r^2 \cdot \frac{\ddot{x}}{r} = r S_2 - r S_1 + N_0$$

$$-\frac{1}{2} \ddot{x} = \frac{S_2}{m_1} - \frac{S_1}{m_1} + \frac{N_0}{r m_1}$$

$$-\frac{1}{2} \ddot{x} = 2 (g + \ddot{x}) - (g - \ddot{x}) + \frac{N_0}{r m_1}$$

$$\ddot{x} \left(-\frac{1}{2} - 2 - 1 \right) = g + \frac{N_0}{r m_1}$$

$$-\frac{7}{2} \ddot{x} = g + \frac{N_0}{r m_1} \Rightarrow \underline{\ddot{x} = -\frac{2}{7} \left(g + \frac{N_0}{r m_1} \right)}$$