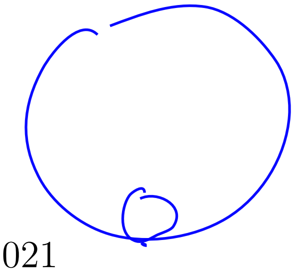
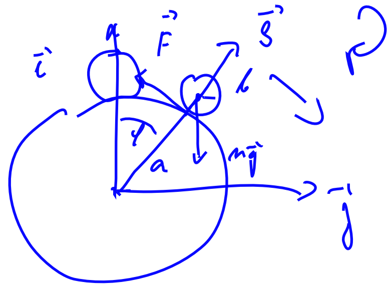


Vaje 5. januar 2021



1. Z vrha temena mirujoče krogle se pod vplivom sile teže skotali kroglica. Določi položaj kje kroglica zapusti kroglo.

$$m\vec a_* = m\vec g + \vec S + \vec F$$

$$m\left(-(a+b)\dot\varphi^2 \vec e_r + (a+b)\ddot\varphi\, \vec e_\varphi\right) = S\vec e_r - F\vec e_\varphi - mg\cos\varphi\, \vec e_r + mg\sin\varphi\, \vec e_\varphi$$

$$m\vec g = mg(-\vec i) = -mg\left((\vec i\cdot\vec e_r)\vec e_r + (\vec i\cdot \vec e_\varphi)\vec e_\varphi\right) =$$

$$= -mg(\cos\varphi\,\vec e_r - \sin\varphi\,\vec e_\varphi)$$

$$\begin{cases} -m(a+b)\dot\varphi^2 = S - mg\cos\varphi \\ m(a+b)\ddot\varphi = -F + mg\sin\varphi \end{cases}$$

$$\frac25 mb^2\ddot\vartheta = \vec N(P_*)\cdot\vec k = \left((-b\vec e_r)\times(-F\vec e_\varphi)\right)\cdot\vec k = bF$$

$$\vec 0 = \vec v_* + \vec\omega\times(-b\vec e_r) = \vec v_* + \dot\vartheta\vec k\times(-b\vec e_r) = (a+b)\dot\varphi\,\vec e_\varphi - b\dot\vartheta\,\vec e_\varphi$$

$$(a+b)\dot\varphi = b\dot\vartheta \Rightarrow \ddot\vartheta = \frac{a+b}{b}\ddot\varphi$$

$$\frac25 m b(a+b)\ddot\varphi = bF \Rightarrow F = \frac25 m(a+b)\ddot\varphi$$

$$\dot\varphi(t=0) = \frac{v_0}{a+b}$$

$$\frac75 m(a+b)\ddot\varphi = mg\sin\varphi \quad /\dot\varphi$$

$$\frac{7}{10}(a+b)\dot\varphi^2 = -g\cos\varphi + C$$

$t=0 \quad \dot\varphi=0;\ \varphi=0$

$C = g$

$$(a+b)\dot\varphi^2 = \frac{10}{7}g(1-\cos\varphi)$$

$$-\frac{10}{7}g(1-\cos\varphi)m = S - mg\cos\varphi$$

$$S=0 \Rightarrow \frac{10}{7}(1-\cos\varphi)=\cos\varphi$$

$$\frac{10}{7} = \frac{17}{7}\cos\varphi$$

$$\boxed{F < kS}$$

$$\boxed{\cos\varphi = \frac{10}{17}} \qquad \boxed{\varphi_0 = \arccos\frac{10}{17}}$$

$$\dot\varphi^2 = \frac{10g}{7(a+b)}2\sin^2\frac\varphi2 \; ; \; \frac{d\varphi}{dt} = \dot\varphi = \underbrace{\sqrt{\frac{20g}{7(a+b)}}}_{A}\sin\frac\varphi2$$

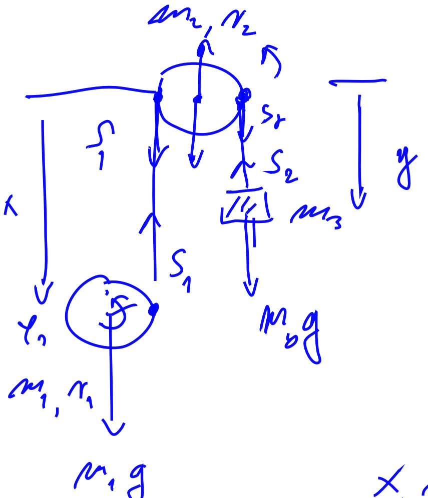
$\varphi_1 \qquad t_0$

$$\infty = \int_0 \frac{d\varphi}{\sin\frac{\varphi}{2}} = \int_0 A\,dt = At_0$$

$\parallel$

ne konvergira

2. Jojo.



$$m_1 \ddot{x} = m_1 g - S_1$$
$$\frac{1}{2} m_1 r_1^2 \ddot{\varphi}_1 = r_1 S_1$$
$$-\frac{1}{2} m_2 r_2^2 \ddot{\varphi}_2 = r_2 (S_1 - S_2)$$
$$m_3 \ddot{y} = m_3 g - S_2$$

$$x, y, S_1, S_2, \varphi_1, \varphi_2$$

$$\dot{x} - r_1 \dot{\varphi}_1 = r_2 \dot{\varphi}_2 = -\dot{y}$$

$$\dot{x} + \dot{y} = r_1 \dot{\varphi}_1$$

$$\dot{y} = -r_2 \dot{\varphi}_2$$

$$l = x + y + \pi r_2 = l_0 + r_1 \varphi_1 \Rightarrow \dot{x} + \dot{y} = r_1 \dot{\varphi}_1$$

$$-\frac{1}{2} m_2 \ddot{y} = S_1 - S_2 \quad ; \quad S_2 = m_3 g - m_3 \ddot{y}$$

$$-\frac{1}{2} m_2 \ddot{y} = S_1 - m_3 g + m_3 \ddot{y} \Rightarrow \left(-\frac{1}{2} m_2 - m_3\right) \ddot{y} = S_1 - m_3 g$$

$$\frac{1}{2} m_1 (\ddot{x} + \ddot{y}) = S_1 = m_1 g - m_1 \ddot{x}$$

$$\frac{3}{2} m_1 \ddot{x} + \frac{1}{2} m_1 \ddot{y} = m_1 g \qquad / \cdot \frac{2}{3}$$

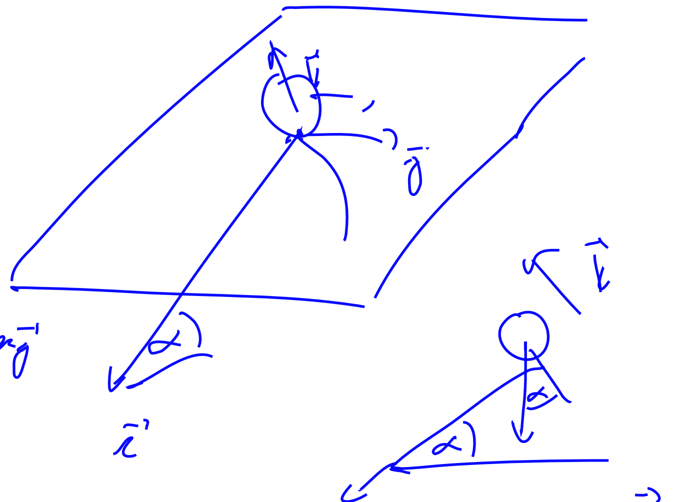
$$+ \left(\frac{1}{2} m_2 + m_3\right) \ddot{y} = -m_1 g + m_1 \ddot{x} + m_3 g$$

$$-m_1 \ddot{x} + \left(\frac{1}{2} m_2 + m_3\right) \ddot{y} = g (m_3 - m_1)$$

$$\left(\frac{1}{3} m_1 + \frac{1}{2} m_2 + m_3\right) \ddot{y} = g \left(m_3 - \frac{1}{3} m_1\right)$$

$$\ddot{y} = \frac{g \left(m_3 - \frac{1}{3} m_1\right)}{\frac{1}{3} m_1 + \frac{1}{2} m_2 + m_3} \qquad m_3 > \frac{1}{3} m_1$$

3. Gibanje krogle po nagnjeni ravnini.



$$m(\ddot{x}\vec{i} + \ddot{y}\vec{j}) = S\vec{k} + F_1\vec{i} + F_2\vec{j} + m\vec{g}$$

$$m\vec{g} = mg(\sin\alpha\,\vec{i} - \cos\alpha\,\vec{k})$$

$$\left.\begin{aligned} m\ddot{x} &= mg\sin\alpha + F_1 \\ m\ddot{y} &= F_2 \\ 0 &= S - mg\cos\alpha \end{aligned}\right]$$

$$\underline{\underline{J}} = \frac{2}{5}mr^2\,\underline{\underline{I}} \qquad \underline{\underline{J}}_*\dot{\vec{\omega}} = \vec{N}_*$$

$$\vec{N}_* = (-r\vec{k}) \times (F_1\vec{i} + F_2\vec{j}) = -rF_1\vec{j} + rF_2\vec{i}$$

$$\frac{2}{5}mr^2\dot{\omega}_1 = rF_2, \quad \frac{2}{5}mr^2\dot{\omega}_2 = -rF_1, \quad \frac{2}{5}mr^2\dot{\omega}_3 = 0$$

$$\vec{0} = \vec{v}_* + \vec{\omega} \times (-r\vec{k}) = \dot{x}\vec{i} + \dot{y}\vec{j} + (\omega_1\vec{i} + \omega_2\vec{j} + \omega_3\vec{k}) \times (-r\vec{k}) =$$

$$= r\omega_1\vec{j} - r\omega_2\vec{i} + \dot{x}\vec{i} + \dot{y}\vec{j} \Rightarrow \dot{x} = r\omega_2, \ \dot{y} = -r\omega_1$$

$$-\frac{2}{5}m\ddot{y} = F_2, \quad \frac{2}{5}m\ddot{x} = -F_1$$

$$m\frac{7}{5}\ddot{x} = mg\sin\alpha; \quad m\frac{7}{5}\ddot{y} = 0$$

$$x(t=0) = 0 = y(t=0)$$
$$\dot{x}(t=0) = v_1 \quad \dot{y}(t=0) = v_2$$

$$\left.\begin{aligned} x &= \frac{5}{14}g\sin\alpha\,t^2 + v_1 t \\ y &= v_2 t \end{aligned}\right\} \Rightarrow x = \frac{5}{14v_2^2}g\sin\alpha\,y^2 + \frac{v_1}{v_2}y$$

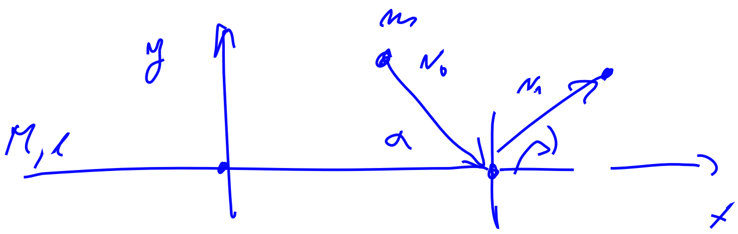
Tir je parabola $v_2 \neq 0$

4. Trk točke v gladko palico.

$2 ; 1, 1 = 4$

$v_1, \beta, v_1^*, v_2^*, \omega$

$v_2^* = 0$



$$\boxed{M = 4m}$$

$$m v_0 \cos\alpha = m v_1 \cos\beta$$

$$-m v_0 \sin\alpha = m v_1 \sin\beta + M v^*$$

$$-m \frac{l}{2} v_0 \sin\alpha = m \frac{l}{2} v_1 \sin\beta + J_* \omega \quad ;$$

$$\frac{1}{2} m v_0^2 = \frac{1}{2} m v_1^2 + \frac{1}{2} J \omega^2 + \frac{1}{2} M v_*^2$$

$$J_* = \frac{1}{12} M l^2$$

$$\rho \int_{-l/2}^{l/2} x^2 dx = J$$

$$\left\{\begin{array}{l} (v_0 \cos\alpha = v_1 \cos\beta)^2 \\ (v_0 \sin\alpha = v_1 \sin\beta + 4 v^*)^2 \\ v_0^2 = v_1^2 + \frac{1}{3} l^2 \omega^2 + 4 v_*^2 \\ -v_0 \sin\alpha = v_1 \sin\beta + \frac{2}{3} l \omega \end{array}\right.$$

$$4 v^* = \frac{2}{3} l \omega$$

$$\omega = 6 v^* / l$$

$$v_0^2 = v_1^2 + \frac{1}{3} l^2 \cdot \frac{36 v^{*2}}{l^2} + 4 v_*^2 = \underline{v_1^2 + 12 v_*^2} + 4 v_*^2 = v_1^2 + 16 v_*^2$$

$$v_0^2 = \underline{v_1^2 + 8 v_1 v_* \sin\beta + 16 v_*^2}$$

$$0 = \underline{8 v_1 v_* \sin\beta} + \cancel{4 v_*^2}$$

$v_1 = 0$ ni mogoče

$v_* = 0$ točka gre skozi palico

$\beta = 0$ iskana rešitev

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