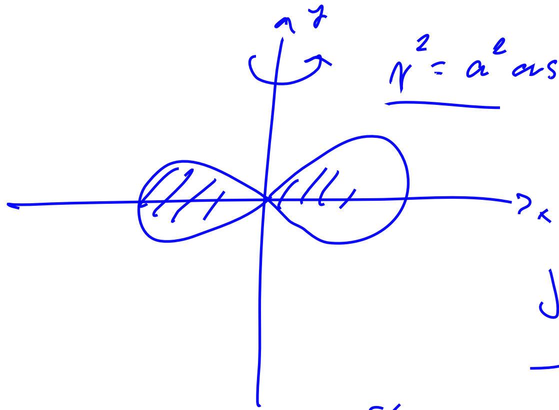
Vaje 12. januar 2021



$$r^2 = a^2\cos 2\vartheta$$

$$J\ddot{\varphi} = N$$

1. Vetrnica.

$$A = 2\int_{-\pi/4}^{\pi/4} \frac{1}{2} r^2\, d\vartheta = 2\int_0^{\pi/4} a^2\cos 2\vartheta = 2a^2\,\frac{1}{2}\sin 2\vartheta\Big|_0^{\pi/4} = \underline{\underline{a^2}}$$

$$J_y = 2\rho\int_{-\pi/4}^{\pi/4} x^2\, dA = 2\rho\int_0^{\pi/4} r^2\cos^2\vartheta\cdot r^2\, d\vartheta = 2\rho\int_0^{\pi/4} r^4\cos^2\vartheta\, d\vartheta =$$

$$x = r\cos\vartheta \qquad = 2a^4\rho\underbrace{\int_0^{\pi/4}\cos^2 2\vartheta\cos^2\vartheta\, d\vartheta}$$

$$\cos^2\vartheta = \frac{1}{2}(1+\cos 2\vartheta)$$

$$J_y = a^4\rho\int_0^{\pi/4}(\cos^2 2\vartheta + \cos^3 2\vartheta)\, d\vartheta$$

$$\int_0^{\pi/4}\cos^2 2\vartheta\, d\vartheta = \int_0^{\pi/4}\sin^2 2\vartheta\, d\vartheta = \frac{1}{2}\cdot\frac{\pi}{4}$$

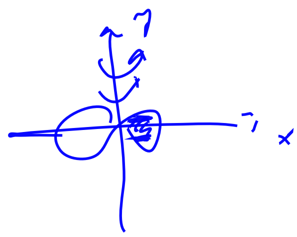
$$\int_0^{\pi/4}\cos^3 2\vartheta\, d\vartheta = \int_0^{\pi/4}(1-\sin^2 2\vartheta)\cdot\cos 2\vartheta\, d\vartheta = \frac{1}{2}\sin 2\vartheta\Big|_0^{\pi/4} - \frac{1}{6}\sin^3 2\vartheta\Big|_0^{\pi/4} =$$

$$= \frac{1}{2} - \frac{1}{6} = \frac{1}{3}$$

$$J_y = a^4\rho\left(\frac{\pi}{8} + \frac{1}{3}\right) = \underline{\underline{\left(\frac{1}{3} + \frac{\pi}{8}\right) m a^2}}$$

$$\vec{N} = \int_A d\vec{N} = 2\cdot\frac{1}{2}\int_{-\pi/4}^{\pi/4}\underline{(\vec{r}\times\vec{f})}\, r^2\, d\vartheta = \int_{-\pi/4}^{\pi/4}(\underline{\vec{r}\times\vec{f}})\, r^2\, d\vartheta$$

$$d\vec{N} = \vec{r}\times\vec{f}\, dA = \vec{r}\times\vec{f}\cdot\frac{1}{2}r^2\, d\vartheta$$



$$\vec{r} = r\vec{e}_r\ ; \quad \vec{f} = f\vec{k} = \mu r^p\vec{k}$$

$$\vec{r}\times\vec{f} = r\mu r^p\,\vec{e}_r\times\vec{k} = -r\mu r^p\,\vec{e}_\varphi$$

$$\vec{e}_\varphi = \underline{-\sin\vartheta}\,\vec{i} + \underline{\cos\vartheta}\,\vec{j}$$

$$= -2\int_0^{\pi/4} r\mu v^p \cos\vartheta\, r^2\, d\vartheta\, \vec{j} = -2\mu\int_0^{\pi/4} r^{p+3}\cos^{p+1}\vartheta\, d\vartheta\, \vec{j}\, \dot{\varphi}^p =$$

$$v^p;\quad v = x\dot{\varphi} = r\cos\vartheta \qquad r^2 = a^2\cos 2\vartheta$$

$$= -2\mu\, a^{\frac{p+3}{2}}\int_0^{\pi/4}\cos^{\frac{p+3}{2}} 2\vartheta\, \cos^{p+1}\vartheta\, d\vartheta\, \dot{\varphi}^p\, \vec{j}$$

$p=1$ linearni upor; $p=2$ kvadratni upor

$$\cos^2\vartheta = \frac{1}{2}(1+\cos 2\vartheta)$$

$\underline{p=1}$; $\quad -2\mu\, a^2\dot{\varphi}\underbrace{\int_0^{\pi/4}\cos^2 2\vartheta\,\cos^2\vartheta\, d\vartheta}\, \vec{j} =$

$$= -\left(\frac{\pi}{8}+\frac{1}{3}\right)a^2\dot{\varphi}\,\vec{j}\,\mu$$

$$\underline{J_y}\,\ddot{\varphi} = \underline{N} \Rightarrow \left(\frac{\pi}{8}+\frac{1}{3}\right)m\,a^2\ddot{\varphi} = -\left(\frac{\pi}{8}+\frac{1}{3}\right)\mu\, a^2\dot{\varphi}$$

$$\ddot{\varphi} = -\frac{\mu}{m}\dot{\varphi}$$

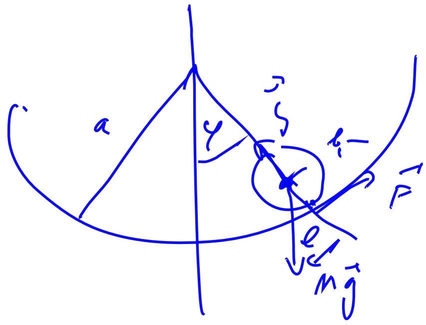
$\dot{\varphi} = u$; $\qquad \dot{u} = -\frac{\mu}{m}u \Rightarrow u = C e^{-\frac{\mu}{m}t}$

$t=0 \Rightarrow \dot{\varphi}(t=0) = \omega_0 \qquad \underline{\dot{\varphi} = \omega_0 e^{-\frac{\mu}{m}t}}$

$\dot{\varphi}(t=t_1) = \frac{1}{2}\omega_0 \Rightarrow \frac{1}{2}\omega_0 = \omega_0 e^{-\frac{\mu}{m}t_1}$

$$-\log 2 = \log\frac{1}{2} = -\frac{\mu}{m}t \Rightarrow \underline{\underline{t = \frac{m\log 2}{\mu}}}$$

2. Kotaljenje krogle v krogli.



$$m\vec{a}_* = m\vec{g} + \vec{S} + \vec{F}$$

$$m\left(-(a-b)\dot{\varphi}^2\vec{e}_r + (a-b)\ddot{\varphi}\vec{e}_\varphi\right) = mg(\cos\varphi\,\vec{e}_r - \sin\varphi\,\vec{e}_\varphi) - S\vec{e}_r + F\vec{e}_\varphi$$

$$-m(a-b)\dot{\varphi}^2 = mg\cos\varphi - S$$

$$\to m(a-b)\ddot{\varphi} = -mg\sin\varphi + F$$

$$J\ddot{\vartheta} = N \quad ; \quad \frac{2}{5}mb^2\ddot{\vartheta}\vec{k} = b\vec{e}_r \times F\vec{e}_\varphi = bF\vec{k}$$

Pogoj kotaljenja

$$\vec{0} = (a-b)\dot{\varphi}\vec{e}_\varphi + \vec{\omega}\times b\vec{e}_r = (a-b)\dot{\varphi}\vec{e}_\varphi + \dot{\vartheta}\vec{k}\times b\vec{e}_r =$$

$$= (a-b)\dot{\varphi}\vec{e}_\varphi + b\dot{\vartheta}\vec{e}_\varphi$$

$$\dot{\vartheta} = -\frac{a-b}{b}\dot{\varphi}$$

$$-\frac{2}{5}mb^2\frac{a-b}{b}\cdot\ddot{\varphi} = bF \Rightarrow F = -\frac{2}{5}m(a-b)\ddot{\varphi}$$

$$\frac{7}{5}m(a-b)\ddot{\varphi} = -mg\sin\varphi \quad /\dot{\varphi} \Rightarrow \frac{7}{10}m(a-b)\dot{\varphi}^2 = mg\cos\varphi + C$$

$$\varphi = \varphi_0 \Rightarrow \dot{\varphi} = 0 \Rightarrow C = -mg\cos\varphi_0$$

$$\boxed{\frac{7}{10}m(a-b)\dot{\varphi}^2 - mg\cos\varphi = -mg\cos\varphi_0}$$

$$T = 2\pi\sqrt{\frac{m}{U''(s_0)}} \quad \Big| \quad \frac{1}{2}m\dot{s}^2 + U(s) = E_0 \qquad s = (a-b)\varphi$$

$$\frac{1}{2}m\left(\frac{7}{5}(a-b)^2\dot{\varphi}^2\right) - mg(a-b)\cos\left(\frac{s}{a-b}\right) = -mg(a-b)\cos\varphi_0 = E_0$$

$$\frac{1}{2}m\dot{s}^2 - \frac{5}{7}mg(a-b)\cos\frac{s}{a-b} = \tilde{E}_0$$

$$U = -\frac{5}{7}mg(a-b)\cos\frac{s}{a-b}$$

$$U' = \frac{5}{7}mg\sin\frac{s}{a-b} \qquad s = s_0 = 0\ ; \quad U'' = \frac{5}{7}mg\cos\frac{s}{a-b}\,\frac{1}{a-b}$$

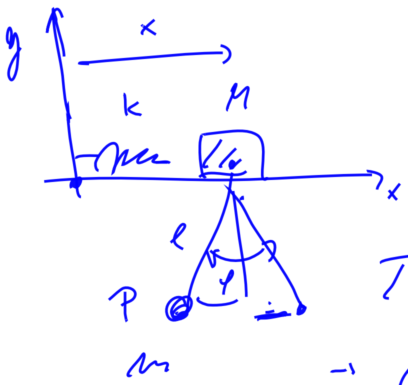
$$U''(s=0) = \frac{5mg}{7(a-b)} \qquad T = 2\pi\sqrt{\frac{7(a-b)}{5g}}$$

$$K + U = E_0$$

$$\frac{1}{2} m v_t^2 + \frac{1}{2} J\omega^2 + U = E_0$$

$$v_t = (a-b)\dot\varphi \qquad \omega = \dot\vartheta$$

3. Nihalo na vzmeti.



Klada je gibljiva brez trenja.

$$T = \frac{1}{2} M\dot x^2 + \frac{1}{2} m v^2$$

$$\vec v = (\dot x + l\dot\varphi\cos\varphi)\vec i + l\dot\varphi\sin\varphi\,\vec j$$

$$P = (x + l\sin\varphi,\ -l\cos\varphi)$$

$$v^2 = (\dot x + l\dot\varphi\cos\varphi)^2 + (l\dot\varphi\sin\varphi)^2 = \dot x^2 + l^2\dot\varphi^2 + 2l\dot x\dot\varphi\cos\varphi$$

$$T = \frac{1}{2}(m+M)\dot x^2 + \frac{1}{2} m l^2\dot\varphi^2 + m l\dot x\dot\varphi\cos\varphi$$

$$U = -mgl\cos\varphi + \frac{1}{2}kx^2$$

$$L = T - U = \frac{1}{2}(m+M)\dot x^2 + \frac{1}{2} m l^2\dot\varphi^2 + m l\dot x\dot\varphi\cos\varphi + mgl\cos\varphi - \frac{1}{2}kx^2$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot x} - \frac{\partial L}{\partial x} = (m+M)\ddot x + (ml\dot\varphi\underbrace{\cos\varphi}_{1})^{\cdot} + kx = 0$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot\varphi} - \frac{\partial L}{\partial \varphi} = ml^2\ddot\varphi + (ml\dot x\underbrace{\cos\varphi}_{1})^{\cdot} - mgl(-\sin\varphi) = 0$$

$$\left[\begin{array}{l} (m+M)\ddot x + ml\ddot\varphi\cos\varphi - \cancel{ml\dot\varphi^2\sin\varphi} + kx = 0 \\ ml^2\ddot\varphi + ml\ddot x\cos\varphi - \cancel{ml\dot x\dot\varphi\sin\varphi} + mgl\sin\varphi = 0 \end{array}\right]$$

$$x = 0,\ \varphi = 0; \qquad \frac{\partial U}{\partial x} = kx; \qquad \frac{\partial U}{\partial \varphi} = mgl\sin\varphi \qquad \operatorname{grad} U\Big|_{x=0,\varphi=0} = 0$$

$$\frac{\partial^2 U}{\partial x^2} = k; \qquad \frac{\partial^2 U}{\partial \varphi^2} = mgl\cos\varphi$$

$$U = \frac{1}{2}\left(x^2 k + mgl\, \varphi^2\right)$$