Vaje 20. oktober 2020

$x = a\,ch\,u \cos v_0$

$u = \operatorname{arch} \frac{x}{a \cos v_0}$

1. Za eliptične koordinate $x = a\cosh u \cos v$, $y = a \sinh u \sin v$:
   - določi koordinatne krivulje; ✓ $v = v_0$; $\boxed{x = x(u),\ y = y(u)}$
   - krivočrtne bazne vektorje;
   - Christoffelove simbole.

$$\frac{x^2}{(a\cos v_0)^2} - \frac{y^2}{(a \sin v_0)^2} = ch^2 u - sh^2 u = 1$$

$u = u_0$

$A = a \cos v_0 \qquad B = a \sin v_0$

$$\boxed{\frac{x^2}{A^2} - \frac{y^2}{B^2} = 1}$$

$\vec{g}_u = \frac{\partial \vec{r}}{\partial u}$ je tangenta na KK $v = konst.$

$y = \sqrt{\frac{x^2}{a^2} - 1}$

$\vec{r} = x\vec{i} + y\vec{j}$

$\vec{g}_u = a\, sh\, u \cos v\, \vec{i} + a\, ch\, u \sin v\, \vec{j}$

$\vec{g}_v = \frac{\partial \vec{r}}{\partial v} = -a\, ch\, u \sin v\, \vec{i} + a\, sh\, u \cos v\, \vec{j}$

$0 = \vec{g}_u \cdot \vec{g}_v = a^2(-sh\,u\, ch\, u \cos v \sin v + sh\,u\, ch\,u \cos v \sin v) = 0$

$\vec{g}_u \cdot \vec{g}_u = a^2(sh^2 u \cos^2 v + ch^2 u \sin^2 v) = a^2(sh^2 u + \sin^2 v(ch^2 u - sh^2 u))$

$\cos^2 v = 1 - \sin^2 v$

$|\vec{g}_u|^2 = a^2(sh^2 u + \sin^2 v) = |\vec{g}_v|^2$

$$\boxed{\frac{\partial \vec{g}_u}{\partial u} = \alpha \vec{g}_u + \beta \vec{g}_v}$$

$\alpha = \Gamma^u_{uu} \qquad \beta = \Gamma^v_{uu}$

$$\frac{\partial \vec{g}_u}{\partial v} = \frac{\partial}{\partial v}\frac{\partial \vec{r}}{\partial u} = \frac{\partial^2 \vec{r}}{\partial v \partial u} = \frac{\partial \vec{r}}{\partial u \partial v} = \frac{\partial \vec{g}_v}{\partial u}$$

$\Gamma^i_{jk} = \Gamma^i_{kj}$

$\frac{\partial \vec{g}_u}{\partial u} = a\, ch\, u \cos v\, \vec{i} + a\, sh\, u \sin v\, \vec{j} \quad / \cdot \vec{g}_u$

$\frac{\partial \vec{g}_u}{\partial u} \cdot \vec{g}_u = \alpha |\vec{g}_u|^2$

$\alpha = \frac{1}{|\vec{g}_u|^2} \frac{\partial \vec{g}_u}{\partial u} \cdot \vec{g}_u = \frac{1}{|\ |^2}(a^2 sh\,u\, ch\,u \cos^2 v + a^2 sh\,u\, ch\,u \sin^2 v) =$

$= \frac{1}{sh^2 u + \sin^2 v} sh\,u\, ch\,u = \frac{sh\,2u}{2(sh^2 u + \sin^2 v)}$

$\frac{\partial \vec{g}_u}{\partial u} \cdot \vec{g}_v = \beta |\vec{g}_v|^2$

$\beta = \ldots \qquad = \Gamma^v_{uu}$

DN Γ^u_{vv}, Γ^v_{vv}, Γ^u_{uv}, Γ^v_{uv} $\quad \left(\frac{\partial \vec{g}_v}{\partial v}\right)$

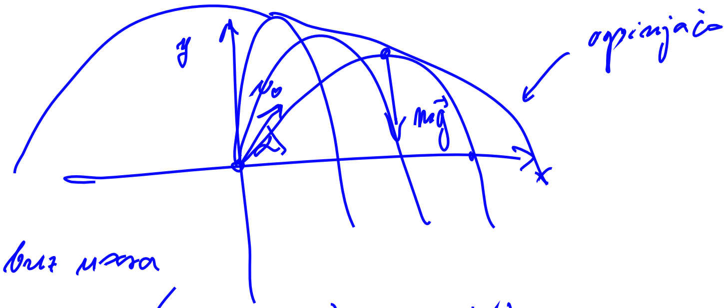
$\vec{a} = a_u \vec{g}_u + a_v \vec{g}_v$

$a_u = \ddot{u} + \Gamma^u_{uu}\dot{u}^2 + \Gamma^u_{vv}\dot{v}^2 + 2\Gamma^u_{uv}\dot{u}\dot{v}$

$u \leftrightarrow 1$

$v \leftrightarrow 2$

$$\frac{\partial \vec{g}_i}{\partial q_j} = \Gamma^{k}_{ij} \vec{g}_k = \sum_{k=1}^{3} \Gamma^k_{ij} \vec{g}_k$$



2. Za poševni met: brez upora

- določi trajektorijo gibanja; ✓ $x = x(t), \quad y = y(t)$
- tir gibanja; $y = y(x)$
- metni prostor.

$$x(t=0) = y(t=0) = 0$$
$$\dot{x}(t=0) = v_0 \cos\alpha$$
$$\dot{y}(t=0) = v_0 \sin\alpha .$$

$$m\vec{a} = \vec{F} = m\vec{g} = -mg\vec{j}$$

$$\vec{a} = \ddot{x}\vec{i} + \ddot{y}\vec{j}$$

$$m\ddot{x} = 0 \Rightarrow \dot{x} = \text{konst} = v_0 \cos\alpha$$

$$\Downarrow$$

$$x = v_0 \cos\alpha \, t$$

$$m\ddot{y} = -mg$$

$$\ddot{y} = -g, \quad \dot{y} = -gt + v_0 \sin\alpha, \quad y = -\frac{1}{2}gt^2 + v_0 \sin\alpha \, t$$

$$t = \frac{x}{v_0 \cos\alpha}$$

$$y = -\frac{1}{2} g \frac{x^2}{v_0^2 \cos^2\alpha} + v_0 \sin\alpha \frac{x}{v_0 \cos\alpha} = -\frac{g}{2v_0^2}\left(1 + \mathrm{tg}^2\alpha\right) x^2 + \mathrm{tg}\alpha \cdot x = f(x,\alpha)$$

Izračun ogrinjače $\hat{y}(x)$

v vsaki točki se $\hat{y}(x)$ dotika $f(x,\alpha)$

‖

skupna točka + skupna tangenta

$$\hat{y}(x) = f(x, \alpha(x)) \qquad \hat{y}' = \frac{\partial f}{\partial x}(x, \alpha(x))$$

$$\hat{y}' = \frac{\partial f}{\partial x}(x, \alpha(x)) + \frac{\partial f}{\partial \alpha}\alpha' \Rightarrow \boxed{\frac{\partial f}{\partial \alpha} = 0} + y = f(x,\alpha)$$

$$\frac{\partial f}{\partial \alpha} = -\frac{g}{2v_0^2} 2\,\mathrm{tg}\alpha \cdot \frac{1}{\cos^2\alpha} x^2 + \frac{1}{\cos^2\alpha} x = 0$$

$$\mathrm{tg}\alpha = \frac{v_0^2}{g}\frac{1}{x}$$

$$\hat{y} = -\frac{g}{2v_0^2}\left(1 + \frac{v_0^4}{g^2}\frac{1}{x^2}\right)x^2 + \frac{v_0^2}{g} = -\frac{g}{2v_0^2}x^2 + \frac{1}{2}\frac{v_0^2}{g}$$

Ogrinjača je parabola.

max. višina $\frac{1}{2}\frac{v_0^2}{g}$ Metna dolžina $\hat{y}(x_0) = 0$

$$x_0^2 = \frac{v_0^4}{g^2}; \quad x_0 = \pm\frac{v_0^2}{g}$$

3. Poševni met z linearnim uporom.

$$m\vec{a} = m\vec{g} + \vec{F}_u$$

$\vec{F}_u \parallel \vec{v}$ ✓

$\vec{F}_u \cdot \vec{v} < 0$ ✓

$\vec{F}_u = -k\vec{v}; \quad k > 0$

splošno $\quad \vec{F}_u = -f(v)\dfrac{\vec{v}}{|\vec{v}|} \quad f(v) > 0$

linearni zakon $\quad f(v) = kv \quad ; \quad v = |\vec{v}|$

kvadratni $\quad f(v) = kv^2 \Rightarrow \vec{F}_u = -k|\vec{v}|\vec{v}$

$$m\vec{a} = m\vec{g} - k\vec{v}$$

$\vec{g} = -g\vec{\jmath} \quad \vec{a} = \vec{g} - \frac{k}{m}\vec{v} \qquad \frac{k}{m} = \mu \qquad \ddot{x} = -\mu\sqrt{\dot{x}^2+\dot{y}^2}\,\dot{x}$

$$\begin{cases} \ddot{x} = -\mu\dot{x} \\ \ddot{y} = -g - \mu\dot{y} \end{cases}$$

linearen + separiran

$\ddot{x} = -\mu\dot{x} \qquad u = \dot{x} \qquad \dot{u} = -\mu u \Leftrightarrow u = C_1 e^{-\mu t}$

$u(t=0) = C_1$

$\dot{x}(t=0) = v_0\cos\alpha \qquad \dot{x} = u = v_0\cos\alpha\, e^{-\mu t}$

$x = -\frac{1}{\mu}v_0\cos\alpha\, e^{-\mu t} + C_2$

$x(t=0) = 0 \Rightarrow C_2 = \frac{v_0\cos\alpha}{\mu} \qquad x = \frac{1}{\mu}v_0\cos\alpha\left(1 - e^{-\mu t}\right)$

$x < \frac{1}{\mu}v_0\cos\alpha$

$t = t(x)$

$\boxed{\ddot{y} = -g - \mu\dot{y}} \qquad v = \dot{y}$

$\dot{v} + \mu v = -g \qquad \dot{v}_0 + \mu v_0 = 0 \quad \Big| \quad \dot{v}_1 + \mu v_1 = -g$

$v_0 = C_1 e^{-\mu t} \qquad v_1 = -\frac{g}{\mu}$

$$\left.\begin{aligned} v &= C_1 e^{-\mu t} - \frac{g}{\mu} \\ v(t=0) &= v_0\sin\alpha \end{aligned}\right\} \Rightarrow C_1 - \frac{g}{\mu} = v_0\sin\alpha$$

$$\dot{y} = v = \left(\frac{g}{\mu} + v_0\sin\alpha\right)e^{-\mu t} - \frac{g}{\mu}$$

$y = -\frac{1}{\mu}\left(\frac{g}{\mu} + v_0\sin\alpha\right)e^{-\mu t} - \frac{g}{\mu}t \qquad x = x(t), \ y = y(t)$

$t \gg 1 \qquad \dot{y} \approx -\frac{g}{\mu} \qquad |\dot{y}| < \frac{g}{\mu}$

4. Prosti pad s kvadratnim uporom.

$$\ddot{y} = -g - k\dot{y}^2$$

5. Določi kot poševnega meta tako, da izstrelek zadane tarčo, ki jo spustimo v prosti pad v trenutku sprožitve izstrelka.